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THE ASYMPTOTICALLY REGULARITY OF MAPS AND SEQUENCES IN PARTIAL CONE METRIC SPACES WITH APPLICATION

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ABSTRACT : In present paper, we define asymptotically regular sequences and maps in partial cone metric space and prove some fixed point theorems for such maps. Our results extend the results of [9] in partial cone metric space.

Key words : Partial cone metric space, asymptotically regular sequences and maps, fixed point.

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1. INTRODUCTION

In 1980, Rzepecki [15] introduced a generalized metric by replacing the set of real numbers with a Banach space E in the metric function where P is a normal cone in E with partial order ≤.

Lin [8] considered the notion of cone metric spaces by replacing real numbers with a cone P in the metric function in which it is called a K-metric. Without mentioning the papers of Lin and Rzepecki, in 2007, Huang and Zhang [5] announced the notion of cone metric spaces (CMS) by replacing real numbers with an ordering Banach space. The authors obtained some fixed point theorems for mappings satisfying different contractive conditions. Afterwards several fixed and common fixed point results on cone metric spaces were introduced in (see [1], [2], [12], [13], [16], [19]).

Recently, in 2013, based on the definition of cone metric spaces and partial metric spaces, Sonmez [17] defined a partial cone metric space. The author developed some fixed point theorems in this generalized setting. Very recently, without using the normality of the cone, Malhotra et al. [10] and Jiang and Li [7] extended the results of [17, 18] to θ-complete partial cone metric spaces.

In the present paper, we define asymptotically regular maps and sequences and present some fixed point results for these maps in partial cone metric space.

2. PRELIMINARIES

First, we invite some standard notations and definitions in cone metric spaces and partial cone metric spaces.

A cone P is a subset of a real Banach space E such that

(i) P is closed, nonempty and $P \neq \{0\}$;

(ii) if a, b are nonnegative real numbers and $x, y \in P$, then $ax + by \in P$;

(iii) $P \cap (-P) = \{0\}$.

Given a cone $P \subseteq E$, we define a partial ordering $\leq$ with respect to P by $x \leq y$ if and only if $y - x \in P$. We shall write $x < y$ to indicate that $x \leq y$ but $x \neq y$, while $x \ll y$ will stand for $y - x \in \text{int } P$, int P denotes the interior of P.

The cone P is called normal if there is a number $k > 0$ such that for all $x, y \in E$, $0 \leq x \leq y$ implies $\| x \| \leq k \| y \|$.

The least positive number k satisfying above is called the normal constant of P.

The cone P is called regular if every increasing sequence which is bounded from above is convergent. That is, if $\{x_n\}$ is sequence such that $x_1 \leq x_2 \leq \ldots \leq x_n \leq \ldots \leq y$ for some $y \in E$, then there is $x \in E$ such that $\| x_n - x \| \to 0 (n \to \infty)$. Equivalently, the cone P is regular if and only if every decreasing sequence which is bounded from below is convergent. It is well known that a regular cone is a normal cone.

Definition 2.1. [5] Let X be a nonempty set. Suppose the mapping $d : X \times X \to P$ satisfies

$(d1)$ $0 < d(x, y)$ for all $x, y \in X$ and $d(x, y) = 0$ if and only if $x = y$;

$(d2)$ $d(x, y) = d(y, x)$ for all $x, y \in X$;

$(d3)$ $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called a cone metric on X, and (X, d) is called a cone metric space.

Definition 2.2[17] A partial cone metric on a nonempty set X is a function $p : X \times X \to P$ such that for all $x, y, z \in X$:

(p1) $x = y \Leftrightarrow p(x, x) = p(x, y) = p(y, y)$,

(p2) $\mathbf{0} \leq p(x, x) \leq p(x, y)$,

(p3) $p(x, y) = p(y, x)$,

(p4) $p(x, y) \leq p(x, z) + p(z, y) - p(z, z)$.

A partial cone metric space is a pair (X, p) such that X is a nonempty set and p is a partial cone metric on X. It is clear that, if $p(x, y) = \mathbf{0}$, then from (p1) and (p2)$x = y$. But if $x = y$, $p(x, y)$ may not be $\mathbf{0}$.

A cone metric space is a partial cone metric space. But there are partial cone metric spaces which are not cone metric spaces. The following an example illustrate a partial cone metric space but not a cone metric space.

Example 2.3. [17] Let $E = R^2$, $P = \{(x, y) \in E : x, y \geq 0\}$, $X = R^+$ and $p : X \times X \to P$ is defined by

$$p(x, y) = (\max \{x, y\}, \alpha \max\{x, y\})$$

where $\alpha \geq 0$ is a constant. Then (X, p) is a partial cone metric space which is not a cone metric space.

Theorem 2.4.[17] Any partial cone metric space (X, p) is a topological space.

Theorem 2.5.[17] Let (X, p) be a partial cone metric space and P be a normal cone with normal constant K, then (X, p) is T_0.

Definition 2.6.[17] Let (X, p) be a partial cone metric space. Let $\{x_n\}$ be a sequence in X and $x \in X$. If for every $c \in \text{int}P$, there is N such that for all $n > N$, $p(x_n, x) \ll c + p(x, x)$, then $\{x_n\}$ is said to be convergent and $\{x_n\}$ converges to x, and x is the limit of $\{x_n\}$. We denote this by $\lim_{n \to \infty} x_n = x$ or, $x_n \to x$ ans $n \to \infty$.

Theorem 2.7.[17] Let (X, p) be a partial cone metric space, P be a normal cone with normal constant K. Let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $p(x_n, x) \to p(x, x)$ as $n \to \infty$.

Sonmez [17] also noted that if (X, p) is a partial cone metric space, P be a normal cone with normal constant K and

$$p(x_n, x) \to p(x, x)\ (n \to \infty), \text{ then } p(x_n, x_n) \to p(x, x) \text{ as } n \to \infty.$$

Lemma 2.8.[17] Let $\{x_n\}$ be a sequence in partial cone metric space (X, p). If a point x is the limit of $\{x_n\}$ and $p(y, y) = p(y, x)$ then y is the limit point of $\{x_n\}$.

Definition 2.9. [17] Let (X, p) be a partial cone metric space. $\{x_n\}$ be a sequence in X. $\{x_n\}$ is Cauchy sequence if there is $a \in P$ such that for every $\varepsilon > 0$ there is N such that for all $n, m > N$

$$\| p(x_n, x_m) - a \| < \varepsilon.$$

Definition 2.10.[17] A partial cone metric space (X, p) is said to be complete if every Cauchy sequence in (X, p) is convergent in (X, p).

Theorem 2.11.[17] Let (X, p) be a partial cone metric space. If $\{x_n\}$ is a Cauchy sequence in (X, p), then it is a Cauchy sequence in the cone metric space (X, d).

Proposition 2.12[3] : Let P be a cone in a real Banach space E. If $a \in P$ and $a \leq ka$, for some $k \in [0, 1)$ then $a = \mathbf{0}$.

Definition 2.13[9]: Let (X, p) and (X', p') be a partial cone metric space. Then a function $f : X \to X'$ is said to be continuous at a point $x \in X$ if and only if it is sequentially continuous at x, that is whenever $\{x_n\}$ is convergent to x we have $\{fx_n\}$ is convergent to $f(x)$.

Lemma 2.14[9] : Let (X, p) be a partial cone metric space, P be a normal cone with normal constant K. Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X and suppose that $x_n \to x$, $y_n \to y$ as $n \to \infty$. Then $p(x_n, y_n) \to p(x, y)$ as $n \to \infty$.

Proposition 2.15[6] : Let (X, d) be a cone metric space and P be a cone in a real Banach space E. If $u \leq v$, $v \ll w$ then $u \ll w$.

3. ASYMPTOTICALLY REGULAR SEQUENCES AND MAPS

Here we will define asymptotically regular sequences and maps in partial cone metric spaces.

Definition 3.1: Let (X, p) be a partial cone metric space. A sequence $\{x_n\}$ in X is said to be asymptotically T-regular if $\lim_{n\to\infty} p(x_n, Tx_n) = \mathbf{0}$ or $\lim_{n\to\infty} p(Tx_n, x_n) = \mathbf{0}$.

Example 3.2. Let $E = R^2$, $P = \{(x, y) \in E : x\,y \geq 0\}$, $X = R^+$ and $p : X \times X \to P$ is defined by

$$p(x, y) = [\max\{x, y\}, \alpha \max\{x, y\}] \quad \forall x, y \in X$$

where $\alpha \geq 0$ is a constant. Then (X, p) is a partial cone metric space. Now let T be a self map of X such that $Tx = \frac{x}{2}$ and choose a sequence $\{x_n\}$, $x_n \neq 0$ for any positive integer n, which converges to zero. We deduce that

$$\begin{aligned}
\lim_{n\to\infty} p(x_n, Tx_n) &= \lim_{n\to\infty} (\max\{x_n, Tx_n\}, \alpha \max\{x_n, Tx_n\}) \\
&= \lim_{n\to\infty} \left(\max\left\{x_n, \frac{x_n}{2}\right\}, \alpha \max\left\{x_n, \frac{x_n}{2}\right\}\right) \\
&= \lim_{n\to\infty} (x_n, \alpha x_n) \\
&= (0, 0) \\
&= \mathbf{0}.
\end{aligned}$$

Hence $\{x_n\}$ is an asymptotically T-regular sequence in (X, p).

Definition 3.3: Let (X, p) be a partial cone metric space. A mapping T of X into itself is said to be asymptotically regular at a point x in X if $\lim_{n\to\infty} p(T^n x, T^{n+1}x) = \mathbf{0}$ or $\lim_{n\to\infty} p(T^{n+1}x, T^n x) = \mathbf{0}$ where $T^n x$ denotes the n^{th} *iterate of* T at x.

Example 3.4. Let (X, p) is a partial cone metric space which is defined in example 3.2 and let T be a self map of X such that $Tx = \frac{x}{4}$ where $x \in X$. Then, we have

$$\lim_{n\to\infty} p(T^n x, T^{n+1}x) = \lim_{n\to\infty} \left(\max\{T^n x, T^{n+1}x\}, \alpha \max\{T^n x, T^{n+1}x\}\right)$$

$$= \lim_{n\to\infty}\left(\max\left\{\frac{x}{2^n}, \frac{x}{2^{n+1}}\right\}, \alpha \max\left\{\frac{x}{2^n}, \frac{x}{2^{n+1}}\right\}\right)$$

$$= \lim_{n\to\infty}\left(\frac{x}{2^n}, \alpha\frac{x}{2^n}\right)$$

$$= (0, \alpha 0)$$

$$= 0.$$

Hence T is an asymptotically regular map at all points of X.

4. MAIN RESULTS

As an application of asymptotically regular maps and sequences, we present some fixed point theorems in the partial cone metric spaces.

Theorem 4.1. Let (X, p) be a complete partial cone metric space ant T be a self mapping of X satisfying the inequality.

$$p(Tx, Ty) \leq a_1 p(x, Tx) + a_2 p(y, Ty) + a_3 p(x, Ty) + a_4 p(y, Tx) + a_5 p(x, y) \quad ...(4.1.1)$$

$$\text{for all } x, y \in X \text{ where } a_1, a_2, a_3, a_4, a_5 \geq 0 \text{ and } (a_3 + a_4 + a_5) < 1 \quad ...(4.1.2)$$

If there exists and asymptotically T-regular sequence in X, then T has a unique fixed point.

Proof : Let $\{x_n\}$ be an asymptotically T-regular sequence in X. Then

$$p(x_n, x_m) \leq p(x_n, Tx_n) + p(Tx_n, x_m) - p(Tx_n, Tx_n)$$

$$\leq p(x_n, Tx_n) + p(x_m, Tx_n) - p(Tx_n, Tx_n)$$

$$\leq p(x_n, Tx_n) + p(x_m, Tx_m) + p(Tx_m, Tx_n) - p(Tx_m, Tx_m) - p(Tx_n, Tx_n)$$

$$\leq p(x_n, Tx_n) + p(x_m, Tx_m) + p(Tx_m, Tx_n)$$

$$\leq p(x_n, Tx_n) + p(x_m, Tx_m) + a_1 p(x_m, Tx_m) + a_2 p(x_n, Tx_n) + a_3 p(x_m, Tx_n)$$
$$+ a_4 p(x_n, Tx_m) + a_5 p(x_m, x_n)$$

$$\leq p(x_n, Tx_n) + p(x_m, Tx_m) + a_1p(x_m, Tx_m) + a_2p(x_n, Tx_n) + a_3[p(x_m, x_n) + p(x_n, Tx_n) - p(x_n, x_n)] + a_4[p(x_n, x_m) + p(Tx_m, x_m) - p(x_m, x_m)] + a_5p(x_m, x_n)$$

$$\leq p(x_n, Tx_n) + p(x_m, Tx_m) + a_1p(x_m, Tx_m) + a_2p(x_n, Tx_n) + a_3[p(x_n, x_m) + p(x_n, Tx_n)] + a_4[p(x_n, x_m) + p(Tx_m, x_m)] + a_5p(x_m, x_n)$$

$$= (1 + a_2 + a_3)p(x_n, Tx_n) + (1 + a_1 + a_4)p(x_m, Tx_m) + (a_3 + a_4 + a_5)p(x_n, x_m) \quad \text{[by } p_3]$$

$$\Rightarrow [1 - (a_3 + a_4 + a_5)]p(x_n, x_m) \leq (1 + a_2 + a_3)p(x_n, Tx_n) + (1 + a_1 + a_4)p(x_m, Tx_m)$$

So, $p(x_n, x_m) \leq \dfrac{(1+a_2+a_3)}{[1-(a_3+a_4+a_5)]}p(x_n, Tx_n) + \dfrac{(1+a_1+a_4)}{[1-(a_3+a_4+a_5)]}p(x_m, Tx_m)$

Since $\{x_n\}$ is an asymptotically T-regular sequence and $m > n$. Therefore $p(x_n, Tx_n) \to \mathbf{0}$ and $p(x_m, Tx_m) \to \mathbf{0}$ when $n \to \infty$

This implies that $p(x_n, x_m) \to \mathbf{0}$ as $n, m \to \infty$. Hence $\{x_n\}$ is a Cauchy sequence. By completeness of X, there is $x \in X$ such that $x_n \to x$ as $n \to \infty$. Therefore

$$\lim_{n\to\infty} p(x_n, x) = p(x, x) = \lim_{n\to\infty} p(x_n, x_n) = \mathbf{0}. \qquad ...(4.1.3)$$

Existence of Fixed Point:

Consider, $p(Tx, x) \leq p(Tx, Tx_n) + p(Tx_n, x) - p(Tx_n, Tx_n)$

$$\leq p(Tx, Tx_n) + p(Tx_n, x)$$

$$\leq a_1p(x, Tx) + a_2p(x, Tx_n) + a_3p(x, Tx_n) + a_4p(x_n, Tx) + a_5p(x, x_n) + p(Tx_n, x)$$

$$\leq a_1p(x, Tx) + a_2p(x_n, Tx_n) + a_3[p(x, x_n) + p(x_n, Tx_n) - p(x_n, x_n)] + a_4[p(x_n, x) + p(x, Tx) - p(x, x)] + a_5p(x, x_n) + [p(Tx_n, x_n) + p(x_n, x) - p(x_n, x_n)]$$

$$\leq a_1p(x, Tx) + a_2p(x_n, Tx_n) + a_3[p(x, x_n) + p(x_n, Tx_n)] + a_4[p(x_n, x) + p(x, Tx)] + a_5p(x, x_n) + [p(Tx_n, x_n) + p(x_n, x)]$$

$$= (a_1 + a_4)p(Tx, x) + (1 + a_2 + a_3)p(Tx_n, x_n) + (1 + a_3 + a_4 + a_5)p(x, x_n)$$

So, $p(Tx, x) \leq \dfrac{(1+a_2+a_3)}{1-(a_1+a_4)} p(Tx_n, x_n) + \dfrac{(1+a_3+a_4+a_5)}{1-(a_1+a_4)} p(x, x_n)$

Since $\{x_n\}$ is an asymptotically T-regular sequence and $\{x_n\}$ is a Cauchy sequence in X. Therefore $x_n \to x$ implies that $p(x_n, Tx_n) \to \mathbf{0}$ and $p(x, x_n) \to \mathbf{0}$ as $n \to \infty$ by (4.1.3). So $\|p(Tx, x)\| = \mathbf{0}$.

$\Rightarrow Tx = x$.

Uniqueness : Let z be another fixed point of T.

Then $p(x, z) = p(Tx, Tz)$

$\leq a_1 p(x, Tx) + a_2 p(z, Tz) + a_3 p(x, Tz) + a_4 p(z, Tx) + a_5 p(x, z)$

$= a_1 p(x, x) + a_2 p(z, z) + a_3 p(x, z) + a_4 p(z, x) + a_5 p(x, z)$

So, $p(x, z) \leq (a_3 + a_4 + a_5) p(x, z)$

$\Rightarrow p(x, z) = \mathbf{0}$ [by Prop. 2.12 and $(a_3 + a_4 + a_5) < 1$]

$\Rightarrow x = z$

This completes the proof of the theorem 4.1.

Theorem 4.2. Let (X, p) be a complete partial cone metric space and T a self mapping of X, satisfying the inequality (4.1.1) for all $x, y \in X$ and $a_1, a_2, a_3, a_4, a_5 \geq 0$ and $\max\{(a_1 + a_4), (a_3 + a_4 + a_5)\} < 1$

If T is asymptotically regular at some fixed point x of X, then there exists a unique fixed point of T.

Proof : Let T be an asymptotically reguar at $x_0 \in X$. Consider the sequence $\{T^n x_0\}$ then for all $m, n \geq 1$

$p(T^m x_0, T^n x_0) \leq a_1 p(T^{m-1} x_0, T^m x_0) + a_2 p(T^{n-1} x_0, T^n x_0) + a_3 p(T^{m-1} x_0, T^n x_0)$

$+ a_4 p(T^{n-1} x_0, T^m x_0) + a_5 p(T^{m-1} x_0, T^{n-1} x_0)$

$$\leq a_1 p(T^{m-1}x_0,\ T^m x_0) + a_2 p(T^{n-1}x_0,\ T^n x_0) + a_3[p(T^{m-1}x_0,\ T^m x_0)$$

$$+ p(T^m x_0,\ T^n x_0) - p(T^m x_0,\ T^m x_0)] + a_4[p(T^{n-1}x_0,\ T^n x_0) + p(T^m x_0,\ T^n x_0)$$

$$- p(T^n x_0,\ T^n x_0)] + a_5[p(T^{m-1}x_0,\ T^m x_0) + p(T^{n-1}x_0,\ T^m x_0) - p(T^m x_0,\ T^m x_0)]$$

$$\leq a_1 p(T^{m-1}x_0,\ T^m x_0) + a_2 p(T^{n-1}x_0,\ T^n x_0) + a_3[p(T^{m-1}x_0,\ T^m x_0) + p(T^m x_0,\ T^n x_0)]$$

$$+ a_4[p(T^{n-1}x_0,\ T^n x_0) + p(T^m x_0,\ T^n x_0)] + a_5[p(T^{m-1}x_0,\ T^m x_0) + p(T^{n-1}x_0,\ T^n x_0)$$

$$+ p(T^m x_0,\ T^n x_0) - p(T^m x_0,\ T^m x_0) - p(T^n x_0,\ T^n x_0)]$$

$$\leq a_1 p(T^{m-1}x_0,\ T^m x_0) + a_2 p(T^{n-1}x_0,\ T^n x_0) + a_3[p(T^{m-1}x_0,\ T^m x_0) + p(T^m x_0,\ T^n x_0)]$$

$$+ a_4[p(T^{n-1}x_0,\ T^n x_0) + p(T^m x_0,\ T^n x_0)] + a_5[p(T^{m-1}x_0,\ T^m x_0) + p(T^{n-1}x_0,\ T^n x_0)$$

$$+ p(T^m x_0,\ T^n x_0)]$$

$$= (a_1 + a_3 + a_5)p(T^{m-1}x_0,\ T^m x_0) + (a_2 + a_4 + a_5)p(T^{n-1}x_0,\ T^n x_0)$$

$$+ (a_3 + a_4 + a_5)p(T^m x_0,\ T^n x_0)$$

So, $p(T^m x_0,\ T^n x_0) \leq \dfrac{(a_1 + a_3 + a_5)}{1-(a_3 + a_4 + a_5)} p(T^{m-1}x_0,\ T^m x_0) + \dfrac{(a_2 + a_4 + a_5)}{1-(a_3 + a_4 + a_5)} p(T^{n-1}x_0,\ T^n x_0)$

Since T is an asymptotically regular at x_0, therefore $p(T^{m-1}x_0,\ T^m x_0) \to \mathbf{0}$ and $p(T^{n-1}x_0,\ T^n x_0)$ $\to \mathbf{0}$ as $m,\ n \to \infty$.

This implies that $p(T^m x_0,\ T^n x_0) \to 0$ as $n,\ m \to \infty$. Hence $\{T^n x_0\}$ is a Cauchy sequence in X. By completeness of X, there is $x \in X$ such that $T^n x_0 \to x$ as $n \to \infty$. Therefore

$$\lim_{n\to\infty} p\left(T^n x_0,\ x\right) = p(x,\ x) = \lim_{n\to\infty} p\left(T^n x_0,\ T^n x_0\right) = \mathbf{0}.$$

Therefore, $\{T^n x_0\}$ is a Cauchy sequence in X which is complete space. So, $\{T^n x_0\} \to x \in X$.

Now, we claim that x is a fixed point to T. For this we have,

$$p(Tx,\ x) \leq p(Tx\ T^n x_0) + p(T^n x_0,\ x) - p(T^n x_0,\ T^n x_0)$$

$$\leq a_1 p(x,\ Tx) + a_2 p(T^{n-1}x_0,\ T^n x_0) + a_3 p(x,\ T^n x_0) + a_4 p(T^{n-1}x_0,\ Tx)$$

$$+ a_5 p(x,\ T^{n-1}x_0) + p(T^n x_0,\ x)$$

$$\leq a_1 p(x, Tx) + a_2 p(T^{n-1}x_0, T^n x_0) + a_3 p(x, T^n x_0) + a_4[p(T^{n-1}x_0, T^n x_0) + p(T^n x_0, Tx) - p(T^n x_0, T^n x_0)] + a_5[p(x, T^n x_0) + p(T^{n-1}x_0, T^n x_0) - p(T^n x_0, T^n x_0)] + p(T^n x_0, x)$$

$$\leq a_1 p(x, Tx) + a_2 p(T^{n-1}x_0, T^n x_0) + a_3 p(x, T^n x_0) + a_4[p(T^{n-1}x_0, T^n x_0) + p(T^n x_0, Tx)] + a_5[p(x, T^n x_0) + p(T^{n-1}x_0, T^n x_0)] + p(T^n x_0, x)$$

$p(Tx, x) \leq (a_1 + a_4)\, p(x, Tx)$ [as $n \to \infty$] [Since $\{T^{n-1}x_0\}$ is a subsequence of $\{T^n x_0\}$]

$\Rightarrow p(Tx, x) = \mathbf{0}$ [by Prop 2.12 and as $a_1 + a_4 < 1$]

$\Rightarrow Tx = x$

The uniqueness of the fixed point x follows as in theorem 4.1 using $(a_3 + a_4 + a_5) < 1$. This completes the proof of the theorem 4.2.

The following example demonstrates theorem 4.2.

Example 4.3 : Let (X, p) is a partial cone metric space which is defined in example 3.2 and let T be a self map of X such that $Tx = \frac{x}{2}$ where $x \in X$. Clearly T is an asymptotically regular map at all points of X. If we take $a_1 = a_2 = a_3 = a_4 = 0$ and $a_5 = \frac{1}{2}$. Then the contractive condition (4.1.1) holds trivially good and 0 is the unique fixed point of the map T.

Conclusion : The asymptotically regularity of the mapping T satisfies the Hardy Rogers contraction condition. It is actually a consequence of $\sum_i^5 a_i < 1$. Thus the theorem 4.1 and the theorem 4.2 extend results due to Hardy Rogers [4] in partial cone metric spaces. It is also worth mentioning that our condition on controls constants says that $\sum_{i=1}^5 a_i$ may exceed 1.

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