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SOME CURVATURE PROPERTIES OF *LP*-SASAKIAN MANIFOLDS

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ABSTRACT : The object of the present paper is to study some curvature conditions in *LP*-Sasakian manifolds.

Key words and Phrases : *LP*-Sasakian manifolds, *M*-projective curvature tensor, Pseudo projective curvature tensor, Conformal curvature tensor, Quasi conformal curvature tensor.

Mathematics Subject Classifications (2010). 53C15, 53C25

1. INTRODUCTION

The notion of Lorentzian Para-Sasakian manifolds was introduced by K. Matsumoto [1] in 1989. Then Mihai and Rosca [2] introduced the same notion independently and obtained several results on this manifold. *LP*-Sasakian manifolds have also been studied by Matsumoto and Mihai [3], Mihai et al. [9], Venkatesha and Bagewadi [4], and many others.

On the other hand, Pokhariyal and Mishra [5] have introduced new curvature tensor called a *M*-projective curvature tensor in a Riemannian manifold and studied its properties. Further, Pokhariyal [6] has studied some properties of this curvature tensor in a Sasakian manifold. Chaubey and Ojha [7], Singh et al. [11] and may others geometers have studied this curvature tensor.

In the present paper we study some curvature conditions on *LP*-Sasakian manifolds. The paper is organized as follows : Section 2 consists the basic definitions of Einstein and η-Einstein manifold. Section 3 is about the study of *M*-projective curvature tensor in *LP*-Sasakian manifolds. Section 4 is devoted to the study of an *LP*-Sasakian manifold satisfying $P(\xi, X) \cdot W^* = 0$ and $W^*(\xi, X) \cdot P = 0$. Section 5 deals with properties of conformal curvature tensor satisfying $C(\xi, X) \cdot W^* = 0$ and $W^*(\xi, X) \cdot C = 0$. Finally, we consider *LP*- Sasakian maniolds satisfying $\overline{C}(\xi, X) \cdot W^* = 0$.

2. PRELIMINARIES

An n-dimensional differentiable manifold M^n is called a Lorentzian Para-Sasakian (briefly LP-Sasakian) manifolds ([1], [2]) if it admits a (1, 1) tensor field ϕ, a contravariant vector field ξ, a covariant vector field η and the Lorentzian metric g, which satisfy

$$\phi^2 X = X + \eta(X)\xi, \tag{2.1}$$

$$\eta(\xi) = -1, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y), \tag{2.3}$$

$$g(X, \xi) = \eta(X), \tag{2.4}$$

$$(\nabla_X \phi)Y = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi, \tag{2.5}$$

$$\nabla_X \xi = \phi X, \tag{2.6}$$

for any vector fields X and Y, where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g. It can be easily seen that in an LP-Sasakian manifold, the following relations hold :

$$\phi(\xi) = 0 \quad \eta(\phi X) = 0, \quad rank(\phi) = (n - 1) \tag{2.7}$$

If we put

$$\Phi(X, Y) = g(\phi X, Y), \tag{2.8}$$

for any vectort fields X and Y, then the tensor field $\Phi(X, Y)$ is symmetric (0, 2) tensor field [1]. Also, since the 1 - form η is closed in an LP - Sasakian manifold, we have ([1], [10])

$$(\nabla_X \eta)(Y) = \phi(X, Y), \quad \phi(X, \xi) = 0, \tag{2.9}$$

for any vector fields X and Y.

Let M^n be an n-dimensional LP-Sasakian manifold with structure (ϕ, ξ, η, g) then we have ([3], [10])

$$g(R(X, Y)Z, \xi) = \eta(R(X, Y)Z) = g(Y, Z)\eta(X) - g(X, Z)\eta(Y), \tag{2.10}$$

$$R(\xi, X)Y = -R(X, \xi)Y = g(X, Y)\xi - \eta(Y)X, \tag{2.11}$$

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y, \tag{2.12}$$

$$S(X, \xi) = (n-1)\eta(X), \tag{2.13}$$

$$S(\phi X, \phi Y) = S(X, Y) + (n-1)\eta(X)\eta(Y), \tag{2.14}$$

for any vector fields X, Y, Z, where R is the Riemannian curvature tensor.

An LP-Sasakian manifold $M^n (n > 2)$ is said to be Einstein manifold if its Ricci tensor S is of the form

$$S(X, Y) = kg(X, Y), \tag{2.15}$$

where k is constant.

An LP-Sasakian manifold M^n is said to be an η-Einstein manifold if its Ricci tensor S is of the form

$$S(X, Y) = \alpha g(X, Y) + \beta\eta(X)\eta(Y), \tag{2.16}$$

for arbitrary vector fields X and Y, where α and β are smooth functions.

3. M-PROJECTIVE CURVATURE TENSOR OF *LP*-SASAKIAN MANIFOLDS

In 1971, Pokhariyal and Mishra [5] defined a tensor field W^* on a Riemannian manifold M^n as

$$\begin{aligned} W^*(X, Y)Z &= R(X, Y)Z - \frac{1}{2(n-1)}[S(Y, Z)X \\ &\quad - S(X, Z)Y + g(Y, Z)QX - g(X, Z)QY], \end{aligned} \tag{3.1}$$

for vector fields X, Y and Z, where S is the Ricci tensor of type (0, 2) and Q is the Ricci operator.

Putting $X = \xi$ in equation (3.1) and using equatings (2.2), (2.4), (2.11) and (2.13) we get

$$\begin{aligned} W^*(\xi, Y)Z &= -W^*(Y, \xi)Z = \tfrac{1}{2}[g(Y, Z)\xi - \eta(Z)Y] \\ &\quad - \frac{1}{2(n-1)}[S(Y, Z)\xi - \eta(Z)QY]. \end{aligned} \tag{3.2}$$

Again, putting $Z = \xi$ in the equation (3.1) and using equations (2.4), (2.12) and (2.13), we get

$$W^*(X, Y)\xi = \frac{1}{2}[\eta(Y)X - \eta(X)Y]$$

$$- \frac{1}{2(n-1)}[\eta(Y)QX - \eta(X)QY]. \tag{3.3}$$

Now, taking the inner product of equations (3.1), (3.2) and (3.3) with ξ and using equations (2.2), (2.4) and (2.13), we get

$$\eta(W^*(X, Y)Z) = \frac{1}{2}[g(Y, Z)\eta(X) - g(X, Z)\eta(Y)]$$

$$- \frac{1}{2(n-1)}[S(Y, Z)\eta(X) - S(X, Z)\eta(Y)], \tag{3.4}$$

$$\eta(W^*(\xi, Y)Z) = -\eta(W^*(Y, \xi)Z)$$

$$= -\frac{1}{2}g(Y, Z) + \frac{1}{2(n-1)}S(Y, Z), \tag{3.5}$$

and

$$\eta(W^*(X, Y)\xi) = 0, \tag{3.6}$$

respectively.

Theorem 3.1. *An LP - Sasakian manifold M^n satisfying the condition $R(\xi, X) \cdot W^* = 0$ is an Einstein manifold.*

Proof. Let $R(\xi, X) \cdot W^*(Y, Z)U = 0$. Then we have

$$R(\xi, X)W^*(Y, Z)U - W^*(R(\xi, X)Y, Z)U$$

$$- W^*(Y, R(\xi, X)Z)U - W^*(Y, Z)R(\xi, X)U = 0, \tag{3.7}$$

which on using the equation (2.11), gives

$$g(X, W^*(Y, Z)U)\xi - \eta(W^*(Y, Z)U)X - g(X, Y)W^*(\xi, Z)U$$

$$- g(X, Z)W^*(Y, \xi)U - g(X, U)W^*(Y, Z)\xi + \eta(Y)W^*(X, Z)U$$

$$+ \eta(Z)W^*(Y, X)U + \eta(U)W^*(Y, Z)X. \tag{3.8}$$

Now, taking the inner product of the above equation with ξ and using equations (2.2), (2.4), (2.11), (3.1), (3.4), (3.5) and (3.6), we obtain

$$\begin{aligned} 'R(Y, Z, U, X) &= g(X, Y)g(Z, U) - g(X, Z)g(Y, U) \\ &+ \frac{1}{2}[g(X, Z)\eta(Y)\eta(U) - g(X, Y)\eta(Z)\eta(U)] \\ &+ \frac{1}{2(n-1)}[S(X, Y)\eta(Z)\eta(U) \\ &- S(X, Z)\eta(Y)\eta(U)]. \end{aligned} \tag{3.9}$$

Taking a frame field and contraction over Z and U, we get

$$S(X, Y) = (n - 1)g(X, Y)$$

This shows that M^n is an Einstein manifold.

Theorem 3.2. *If an LP-Sasakian manifold M^n satisfies the condition $W^*(\xi, X). R = 0$, then*

$$S(QX, Y) = (n - 1)^2 g(X, Y).$$

Proof. Let $W^*(\xi, X) \cdot R(Y, Z)U = 0$. Then, we have

$$\begin{aligned} &W^*(\xi, X)R(Y, Z)U - R(W^*(\xi, X)Y, Z)U \\ &- R(Y, W^*(\xi, X)Z)U - R(Y, Z)W^*(\xi, X)U = 0, \end{aligned} \tag{3.10}$$

which on using the equation (3.2), gives

$$\begin{aligned} &g(X, R(Y, Z)U)\xi - \eta(R(Y, Z)U)X - g(X, Y)R(\xi, Z)U \\ &+ \eta(Y)R(X, Z)U - g(X, Z)R(Y, \xi)U + \eta(Z)R(Y, X)U \\ &- g(X, U)R(Y, Z)\xi + \eta(U)R(Y, Z)X - \frac{1}{n-1}[S(X, R(Y, Z)U)\xi \\ &- \eta(R(Y, Z)U)QX - S(X, Y)R(\xi, Z)U + \eta(Y)R(QX, Z)U \\ &- S(X, Z)R(Y, \xi)U + \eta(Z)R(Y, QX)U - S(X, U)R(Y, Z)\xi \\ &+ \eta(U)R(Y, Z)QX] = 0. \end{aligned} \tag{3.11}$$

Now, taking the inner product of the above equation with ξ and using equations (2.2), (2.4), (2.11), (2.12) and (2.13), we obtain

$$g(X, R(Y, Z)U) - g(X, Y)\eta(R(\xi, Z)U) + \eta(Y)\eta(R(X, Z)U)$$
$$- g(X, Z)\eta(R(Y, \xi)U) + \eta(Z)\eta(R(Y, X)U) - g(X, U)\eta(R(Y, Z)\xi)$$
$$+ \eta(U)\eta(R(Y, Z)X) - \frac{1}{n-1}['R(Y, Z, U, QX) - S(X, Y)\eta(R(\xi, Z)U)$$
$$+ \eta(Y)\eta(R(QX, Z)U) - S(X, Z)\eta(R(Y, \xi)U) + \eta(Z)\eta(R(Y, QX)U)$$
$$- S(X, U)\eta(R(Y, Z)\xi) + \eta(U)\eta(R(Y, Z)QX)] = 0. \tag{3.12}$$

Taking a frame field nad contraction over Z and U, we get

$$S(QX, Y) = (n - 1)^2 g(X, Y).$$

This completes the proof.

Theorem 3.3. *If an LP-Sasakian manifold M^n satisfies the condition $W^*(\xi, X). S = 0$, then*

$$S(QX, Y) = -(n - 1)^2 g(X, Y) + 2(n - 1)S(X, Y).$$

Proof. Let $W^*(\xi, X) \cdot S(Y, Z) = 0$. Then, we have

$$S(W^*(\xi, X)Y, Z) + S(Y, W^*(\xi, X)Z) = 0, \tag{3.13}$$

which on using the equation (3.2), gives

$$(n - 1)[g(X, Y)\eta(Z) + g(X, Z)\eta(Y)] - S(X, Z)\eta(Y) - S(X, Y)\eta(Z)$$
$$+ \frac{1}{(n-1)}[S(QX, Y)\eta(Z) - S(QX, Z)\eta(Y)] = 0. \tag{3.14}$$

Now, putting $Z = \xi$ in the above equation and using equations (2.2), (2.4) and (2.13), we get

$$S(QX, Y) = -(n - 1)^2 g(X, Y) + 2(n - 1)S(X, Y).$$

This completes the proof.

4. *LP*-SASAKIAN MANIFOLDS SATISFYING $P(\xi, X) \cdot W^* = 0$ AND $W^*(\xi, X) \cdot P = 0$

Projective curvature tensor P of the manifold M^n is given by [8]

$$P(X, Y)Z = R(X, Y)Z - \frac{1}{n-1}[S(Y, Z)X - S(X, Z)Y]. \tag{4.1}$$

Putting $X = \xi$ in the above equation and the using equations (2.11) and (2.13), we get

$$P(\xi, Y)Z = -P(Y, \xi)Z = g(Y, Z)\xi - \frac{1}{n-1}S(Y, Z)\xi. \tag{4.2}$$

Again, putting $Z = \xi$ in the equation (4.1) and using the equations (2.12) and (2.13), we get

$$P(X, Y)\xi = 0. \tag{4.3}$$

Now, taking the inner product of equations (4.1), (4.2) and (4.3) with ξ, we get

$$\eta(P(X, Y)Z = g(Y, Z)\eta(X) - g(X, Z)\eta(Y)$$
$$- \frac{1}{n-1}[S(Y, Z)\eta(X) - S(X, Z)\eta(Y)], \tag{4.4}$$

$$\eta(P(\xi, Y)Z) = - \eta(P(Y, \xi)Z) = - g(Y, Z) + \frac{1}{n-1}S(Y, Z), \tag{4.5}$$

and

$$\eta(P(X, Y)\xi = 0 \tag{4.6}$$

respectively.

Theorem 4.1. *If an LP-Sasakian manifold M^n satisfies the condition $P(\xi, X). W^* = 0$ then*

$$S(QX, Y) = 2(n - 1)[S(X, Y) - (n - 1)g(X, Y)].$$

Proof. Let $P(\xi, X) \cdot W^*(Y, Z)U = 0$. Then, we have

$$P(\xi, X)W^*(Y, Z)U - W^*(P(\xi, X)Y, Z)U$$
$$- W^*(Y, P(\xi, X)Z)U - W^*(Y, Z)P(\xi, X)U = 0. \tag{4.7}$$

which on using the equation (4.2), gives

$$g(X, W^*(Y, Z)U)\xi - g(X, Y)W^*(\xi, Z)U - g(X, Z)W^*(Y, \xi)U$$
$$- g(X, U)W^*(Y, Z)\xi - \frac{1}{n-1}[S(X, W^*(Y, Z)U)\xi - S(X, Y)W^*(\xi, Z)U$$
$$- S(X, Z)W^*(Y, \xi)U - S(X, U)W^*(Y, Z)\xi] = 0. \tag{4.8}$$

Now, taking the inner product of above equation with ξ and using equation (2.2), (2.4), (3.1), (3.4), (3.5) and (3.6), we obtain

$$\frac{1}{(n-1)}{}'R(Y, Z, U, QX) = {}'R(Y, Z, U, X) + \frac{1}{2}[g(X, Z)g(Y, U)$$

$$- g(X, Y)g(Z, U)] - \frac{1}{2(n-1)^2}[g(Z, U)X(QX, Y)$$

$$- g(Y, U)S(QX, Z)]. \quad (4.9)$$

Taking a frame field and contraction over Z and U, we get

$$S(QX, Y) = 2(n-1)[S(X, Y) - (n-1)g(X, Y)].$$

This completes the proof.

Theorem 4.2. *If a LP-Sasakian manifold M^n satisfies the condition $W^*(\xi, X). P = 0$ then*

$$S(QX, Y) = \frac{n(n-1)^2}{n-2}g(X, Y) + \frac{2n(n-1)}{n-2}S(X, Y)$$

Proof. Let $W^*(\xi, X) \cdot P(Y, Z)U = 0$. Then, we have

$$W^*(\xi, X)P(Y, Z)U - P(W^*(\xi, X)Y, Z)U$$

$$- P(Y, W^*(\xi, X)Z)U - P(Y, Z)W^*(\xi, X)U = 0. \quad (4.10)$$

which on using the equation (3.2), gives

$$g(X, P(Y, Z)U)\xi - \eta(P(Y, Z)U)X - g(X, Y)P(\xi, Z)U + \eta(Y)P(X, Z)U$$

$$- g(X, Z)P(Y, \xi)U + \eta(Z)P(Y, X)U - g(X, U)P(Y, Z)\xi + \eta(U)P(Y, Z)X$$

$$- \frac{1}{n-1}[S(X, P(Y, Z)U)\xi - \eta(P(Y, Z)U)QX - S(X, Y)P(\xi, Z)U$$

$$+ \eta(Y)P(QX, Z)U - S(X, Z)P(Y, \xi)U + \eta(Z)P(Y, QX)U$$

$$- S(X, U)P(Y, Z)\xi + \eta(U)P(Y, Z)QX] = 0. \quad (4.11)$$

Now, takiing the inner product of above equation with ξ and using equations (2.2), (2.4), (4.1), (4.4), (4.5) and (4.6), we obtain

$$- 'R(Y, Z, U, X) + g(X, Y)g(Z, U) - g(X, Z)g(Y, U) + g(X, Z)\eta(Y)\eta(U)$$

$$- g(X, Y)\eta(Z)\eta(U) - \frac{1}{(n-1)}['R(Y, Z, U, QX) + 2S(X, Y)\eta(Z)\eta(U)$$

$$- 2S(X, Z)\eta(Y)\eta(U) + S(X, Z)g(Y, U) - S(X, Y)g(Z, U)]$$

$$+ \frac{1}{(n-1)^2}[S(QX, Z)\eta(Y)\eta(U) - S(QX, Y)\eta(Z)\eta(U)] = 0. \quad (4.12)$$

Taking a frame field and contraction over Z and U, we get

$$S(QX, Y) = -\frac{n(n-1)^2}{n-2}g(X, Y) + \frac{2n(n-1)}{n-2}S(X, Y).$$

This completes the proof.

5. *LP*-SASAKIAN MANIFOLDS SATISFYING $C(\xi, X) \cdot W^* = 0$ AND $W^*(\xi, X) \cdot C = 0$

Conformal curvature tensor C of the manifold M^n is given by [10]

$$C(X, Y)Z = R(X, Y)Z - \frac{1}{(n-2)}[S(Y, Z)X - S(X, Z)Y$$

$$+ g(Y, Z)QX - g(X, Z)QY]$$

$$+ \frac{r}{(n-1)(n-2)}[g(Y, Z)X - g(X, Z)Y]. \quad (5.1)$$

Putting $X = \xi$ in the above equatin and using the equations (2.11) and (2.13), we get

$$C(\xi, Y)Z = -C(Y, \xi)Z = \frac{1+r-n}{(n-1)(n-2)}[g(Y, Z)\xi - \eta(Z)Y]$$

$$- \frac{1}{n-2}[S(Y, Z)\xi - \eta(Z)QY]. \quad (5.2)$$

Again, putting $Z = \xi$ in the equation (5.1) and using the equations (2.12) and (2.13), we get

$$C(X, Y)\xi = \frac{1+r-n}{(n-1)(n-2)}[\eta(Y)X - \eta(X)Y]$$

$$- \frac{1}{n-2}[\eta(Y)QX - \eta(X)QY]. \quad (5.3)$$

Now, taking the inner product of the equations (5.1) and (5.2) with ξ, we get

$$\eta(C(X,\ Y)Z = \frac{1+r-n}{(n-1)(n-2)}[g(Y,\ Z)\eta(X) - g(X,\ Z)\eta(Y)]$$

$$- \frac{1}{n-2}[S(Y,\ Z)\eta(X) - S(X,\ Z)\eta(Y)], \tag{5.4}$$

$$\eta(C(\xi,\ Y)Z) = -\eta(C(Y,\ \xi)Z)$$

$$= \frac{1+r-n}{(n-1)(n-2)}[-g(Y,\ Z) - \eta(Y)\eta(Z)]$$

$$- \frac{1}{n-2}[-S(Y,\ Z) - \eta(Z)\eta(QY)]. \tag{5.5}$$

Theorem 5.1. *If an LP-Sasakian manifold M^n satisfies the condition $C(\xi,\ X).\ W^* = 0$ then*

$$S(QX,\ Y) = \left(\frac{(n^2-3n+r+2)}{(n-1)}\right)S(X,\ Y) + (1+r-n)g(X,\ Y)$$

Proof. Let $C(\xi,\ X)\cdot W^*(Y,\ Z)U = 0$. Then, we have

$$C(\xi,\ X)W^*(Y,\ Z)U - W^*(C\xi,\ X)Y,\ Z)U$$

$$- W^*(Y,\ C(\xi,\ X)Z)U - W^*(Y,\ Z)C(\xi,\ X)U = 0, \tag{5.6}$$

which on using the equation (5.2), gives

$$\frac{1+r-n}{(n-1)(n-2)}[g(X,\ W^*(Y,\ Z)U)\xi - \eta(W^*(Y,\ Z)U)X$$

$$- g(X,\ Y)W^*(\xi,\ Z)U + \eta(Y)W^*(X,\ Z)U - g(X,\ Z)W^*(Y,\ \xi)U$$

$$+ \eta(Z)W^*(Y,\ X)U - g(X,\ U)W^*(Y,\ Z)\xi + \eta(U)W^*(Y,\ Z)X]$$

$$- \frac{1}{n-2}[S(X,\ W^*(Y,\ Z)U)\xi - \eta(W^*(Y,\ Z)U)QX - S(X,\ Y)W^*(\xi,\ Z)U$$

$$+ \eta(Y)W^*(QX,\ Z)U - S(X,\ Z)W^*(Y,\ \xi)U + \eta(Z)W^*(Y,\ QX)U$$

$$- S(X,\ U)W^*(Y,\ Z)\xi + \eta(U)W^*(Y,\ Z)QX] = 0. \tag{5.7}$$

Now, taking the inner product of the above equation with ξ and using the equations (2.2), (2.4), (3.1), (3.4), (3.5) and (3.6), we obtain

$$\frac{1+r-n}{(n-1)(n-2)}[-'R(Y,Z,U,X)+\frac{1}{2(n-1)}\{g(Z,U)S(X,Y)-g(Y,U)S(X,Z)\}$$

$$+\frac{1}{2}\{g(X,Y)g(Z,U)-g(X,Z)g(Y,U)+g(X,Z)\eta(Y)\eta(U)-g(X,Y)\eta(Z)\eta(U)\}$$

$$+\frac{1}{2(n-1)}\{S(X,Y)\eta(Z)\eta(U)-S(Z,X)\eta(Y)\eta(U)\}]$$

$$-\frac{1}{(n-2)}[-'R(Y,Z,U,QX)+\frac{1}{2(n-1)}\{g(Z,U)S(QX,Y)-g(Y,U)S(QX,Z)\}$$

$$+\frac{1}{2}\{S(X,Y)g(Z,U)-S(X,Z)g(Y,U)+S(X,Z)\eta(Y)\eta(U)-S(X,Y)\eta(Z)\eta(U)\}$$

$$+\frac{1}{2(n-1)}\{S(QX,Y)\eta(Z)\eta(U)-S(QX,Z)\eta(Y)\eta(U)\}]=0. \tag{5.8}$$

Taking a frame field and contraction over Z and U, we get

$$S(QX,Y)=\left(\frac{(n^2-3n+r+2)}{(n-1)}\right)S(X,Y)+(1-n+r)g(X,Y).$$

This completes the proof.

Theorem 5.2. *If an LP-Sasakian manifold M^n satisfies the condition $W^*(\xi, X). C = 0$, then the manifold is an Einstein manifold.*

Proof. Let $W^*(\xi, X)\cdot C(Y, Z)U = 0$. Then, we have

$$W^*(\xi,X)C(Y,Z)U-C(W^*(\xi,X)Y,Z)U$$

$$-C(Y,W^*(\xi,X)Z)U-C(Y,Z)W^*(\xi,X)U=0, \tag{5.9}$$

which on using the equation (3.2), gives

$$g(X,C(Y,Z)U)-\eta(C(Y,Z)U)X-g(X,Y)C(\xi,Z)U$$

$$+\eta(Y)C(X,Z)U-g(X,Z)C(Y,\xi)U+\eta(Z)C(Y,X)U$$

...

$$- g(X, U)C(Y, Z)\xi + \eta(U)C(Y, Z)X - \frac{1}{n-1}[S(X, C(Y, Z)U)\xi$$

$$- \eta(C(Y, Z)U)QX - S(X, Y)C(\xi, Z)U + \eta(Y)C(QX, Z)U$$

$$- S(X, Z)C(Y, \xi)U + \eta(Z)C(Y, QX)U - S(X, U)C(Y, Z)\xi$$

$$+ \eta(U)C(Y, Z)QX] = 0. \tag{5.10}$$

Now, taking the inner product of above equation with ξ and using equations (2.2), (2.4), (5.1), (5.4) and (5.5), we obtain

$$- \frac{1}{(n-1)}{}'R(Y, Z, U, QX) = - {}'R(Y, Z, U, X) + \frac{1}{(n-2)}[S(X, Y)g(Z, U)$$

$$- S(X, Z)g(Y, U) + (n-1)\{g(X, Z)\eta(Y)\eta(U) - g(X, Y)\eta(Z)\eta(U)\}$$

$$+ 2\{S(X, Y)\eta(Z)\eta(U) - S(X, Z)\eta(Y)\eta(U)\} + \frac{1}{(n-1)}\{S(QX, Z)g(Y, U)$$

$$- S(QX, Y)g(Z, U) + S(QX, Z)\eta(Y)\eta(U) - S(QX, Y)\eta(Z)\eta(U)\}]$$

$$- \frac{r}{(n-1)(n-2)}[g(Z, U)g(X, Y) - g(Y, U)g(X, Z) + \frac{1}{(n-1)}\{g(Z, U)S(X, Y)$$

$$- g(Y, U)S(X, Z)\}] + \frac{1+r-n}{(n-1)(n-2)}[g(Z, U)g(X, Y) - g(Y, U)g(X, Z)$$

$$+ \frac{1}{(n-1)}\{S(X, Z)g(Y, U) - S(X, Y)g(Z, U)\}]. \tag{5.11}$$

Taking a frame field and contraction over Z and U, we get

$$S(X, Y) = 2rg(X, Y).$$

This completes the proof.

6. *LP*-SASAKIAN MANIFOLDS SATISFYING $\overline{C}(\xi, X) \cdot W^* = 0$

The notion of the quasi-conformal curvature tensor $\overline{C}$ was introduced by Yano and Sawaki [10]. They defined the quasi-conformal curvature tensor by

$$\overline{C}(X, Y)Z = aR(X, Y)Z + b[S(Y, Z)X - S(X, Z)Y + g(Y, Z)QX - g(X, Z)QY] - \frac{r}{n}\left(\frac{a}{n-1} + 2b\right)[g(Y, Z)X - g(X, Z)Y], \tag{6.1}$$

where a and b are constants such that $ab \neq 0$. If $a = 1$ and $b = \frac{1}{n-2}$, then above equation reduces to conformal curvature tensor given by (5.1). Thus the conformal curvature tensor C is a particular case of the Quasi-conformal curvature tensor $\overline{C}$.

Putting $X = \xi$ in equation (6.1) and using the equations (2.11) and (2.13), we get

$$\overline{C}(\xi, Y) = -\overline{C}(Y, \xi)Z) = \left[a + b(n-1) - \frac{r}{n}\left(\frac{a}{n-1} + 2b\right)\right][g(Y, Z)\xi - \eta(Z)Y] + b[S(Y, Z)\xi - \eta(Z)QY]. \tag{6.2}$$

Again, putting $Z = \xi$ in the equation (6.1) and using the equations (2.12) and (2.13), we get

$$\overline{C}(X, Y)\xi = \left[a + b(n-1) - \frac{r}{n}\left(\frac{a}{n-1} + 2b\right)\right][\eta(Y)X - \eta(X)Y] + b[\eta(Y)QX - \eta(X)QY]. \tag{6.3}$$

Now, taking the inner product of equations (6.1), (6.2) and (6.3) with ξ, we get

$$\eta(\overline{C}(X, Y)Z) = \left[a + b(n-1) - \frac{r}{n}\left(\frac{a}{n-1} + 2b\right)\right][g(Y, Z)\eta(X) - g(X, Z)\eta(Y)] + b[\eta(Y)QX - \eta(X)QY], \tag{6.4}$$

$$\begin{aligned}\eta(\overline{C}(\xi, Y)Z) &= -\eta(C(Y, \xi)Z) \\ &= \left[a + b(n-1) - \frac{r}{n}\left(\frac{a}{n-1} + 2b\right)\right][-g(Y, Z) - \eta(Y, Z) - \eta(Y)\eta(Z)] \\ &\quad + b[-S(Y, Z) - \eta(Z)\eta(QY)]\end{aligned} \tag{6.5}$$

and

$$\eta(\overline{C}(X, Y)\xi) = 0 \tag{6.6}$$

respectively.

Theorem 6.1. *If an LP-Sasakian manifold M^n satisfies the condition $\overline{C}(\xi, X)\cdot W^* = 0$ then*

$$S(QX, Y) = \left[(n-1)-\frac{A}{b}\right]S(X, Y)-\left[\frac{2(n-1)+r}{n}\right]\frac{A}{b}\eta(X)\eta(Y)$$

$$+\left[\frac{n(n-1)-r}{n}\right]\frac{A}{b}g(X, Y).$$

where $A = \left[a+b(n-1)-\frac{r}{n}\left(\frac{a}{n-1}+2b\right)\right]$.

Proof. Let $\overline{C}(\xi, X)\cdot W^*(Y, Z)U = 0$. then, we have

$$\overline{C}(\xi, X)W^*(Y, Z)U - W^*(\overline{C}(\xi, X)Y, Z)U$$
$$- W^*(Y, \overline{C}(\xi, X)Z)U - W^*(Y, Z)\overline{C}(\xi, X)U = 0, \tag{6.7}$$

which on using the equation (6.2), gives

$$A[g(X, W^*(Y, Z)U)\xi - \eta(W^*(Y, Z)U)X - g(X, Y)W^*(\xi, Z)U$$
$$+ \eta(Y)W^*(X, Z)U - g(X, Z)W^*(Y, \xi)U + \eta(Z)W^*(Y, X)U$$
$$- g(X, U)W^*(Y, Z)\xi + \eta(U)W^*(Y, Z)X]$$
$$+ b[S(X, W^*(Y, Z)U)\xi + \eta(W^*(Y, Z)U)\eta(X) - S(X, Y)W^*(\xi, Z)U$$
$$+ \eta(Y)W^*(QX, Z)U - S(X, Z)W^*(Y, \xi)U + \eta(Z)W^*(Y, QX)U$$
$$- S(X, U)W^*(Y, Z)\xi + \eta(U)W^*(Y, Z)QX] = 0. \tag{6.8}$$

Now, taking the inner product of above equation with ξ and using equations (2.2), (2.4), (3.1), (3.4), (3.5) and (3.6), we obtain

$$A[-'R(Y, Z, U, X) - \frac{1}{2(n-1)}\{g(Z, U)S(X, Y) - g(Y, U)S(X, Z)$$
$$- 2S(Y, U)\eta(X)\eta(Z) - S(Z, U)g(X, Y) - S(Z, U)\eta(X)\eta(Y) - S(X, Z)\eta(Y)\eta(U)$$

$$+ S(X, Y)\eta(Z)\eta(U)\} + \frac{1}{2}\{g(Z, U)g(X, Y) - g(X, Z)g(Y, U)$$

$$+ g(X, Z)\eta(Y)\eta(U) - g(X, Y)\eta(Z)\eta(U)\}]$$

$$+ b[-'R(Y, Z, U, QX) + \frac{1}{2(n-1)}\{g(Z, U)S(X, QY) - (g(Y, U)S(X, QZ)$$

$$+ S(QX, Y)\eta(Z)\eta(U) - S(QX, Z)\eta(Y)\eta(U)\} + \frac{1}{2}\{g(Z, US(X, Y)$$

$$- g(Y, U)S(X, Z) + S(X, Z)\eta(Y)\eta(U) - S(X, Y)\eta(Z)\eta(U)\}]. \quad (6.9)$$

Taking a frame field and contraction over Z and U, we get

$$S(QX, Y) = \left[(n-1) - \frac{A}{b}\right] S(X, Y) - \left[\frac{2(n-1)+r}{n}\right]\frac{A}{b}\eta(X)\eta(Y)$$

$$+\left[\frac{n(n-1)-r}{n}\right]\frac{A}{b} g(X, Y).$$

This completes the proof.

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