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PROPERTIES OF δ-LORENTZIAN β-KENMOTSU MANIFOLDS

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ABSTRACT : In this paper, we obtain basic results and properties of δ-Lorentzian β-Kenmotsu manifolds. Properties of Ricci semisymmetric and conformally flat δ-Lorentzian β-Kemmotsu manifolds are obtained.

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1. INTRODUCTION

In 1969, T. Takahashi [12] has introduced Sasakian manifolds with pseudo Riemannian metric and showed that one can study the Lorentzian Sasakian structure with an indefinite metric. In 1990, K.L. Duggal [8] has initiated the space time manifolds. S.Y. Perktas, Erol Kilic, M.M. Tripathi and S. Keles [10] and Hakan Oztur, Nasip Aktan and Cengizhan Murathan [9], U.C. De [3], U.C. De and J.B. Jun, Goutam Pathak [4], U.C. De [5], Lovejoy Das, R.N. Singh, Manoj Kumar Pande [6], U.C. de, A. Yildiz, B.E. Acet [7] have studied the various properties of Lorentzian β-Kenmotsu manifolds. Some other authors (see the list [2], [8], [11]) studied Lorentzian β-Kenmotsu manifolds. Recently Vilas Khairnar [15] studied weak symmetries of δ-Lorentzian β-Kenmotsu manifolds.

In Section 2, we consider $(2n+1)$ dimensional differrentiable manifold M with Lorentzian almost contact metric structure with indefinite metric g. In this Section, some background information for defining δ-Lorentzian β-Kenmotsu manifold has been given. Further, various basic results are studied. Concrete Example for the existance of δ-Lorentzian β-Kenmotsu manifold is given. Section 3 is devoted to the study of the generalised recurrent properties of δ-Lorentzian β-Kenmotsu manifolds. This section includes some of the results of Hakan Oztur, Nasip Aktan and Cengizhan Murathan [9] as special cases.

Section 4 deals with the properties of Ricci semisymmetric and semisymmetric δ-Lorentzian β-Kenmotsu manifolds and generalises the results of Hakan Oztur, Nasip Aktan and Cengizhan Murathan [9]. Finally, in Section 5, the properties of confomally flat space δ-Lorentzian β-Menmotsu manifold are obtained.

2. δ - LORENTZIAN β - KENMOTSU MANIFOLD

For an almost Lorentzian contact manifold, we have

$$\varphi^2 X = X + \eta(X)\xi,\ \eta(\xi) = -1,\ \eta(X) = g(X,\ \xi)$$

where φ is a tensor field of type (1,1), ξ is characteristic vector field and η is the 1-form. From these conditions, one can deduce that

$$\varphi(\xi) = 0,\ \eta(\varphi(X)) = 0$$

for any vector field X on M. It is well known that the Lorentzian contact metric structure or Lorentzian Kenmotsu structure [8] satisfies

$$(\nabla_X \varphi)Y = g(\varphi(X),\ Y)\xi + \eta(Y)\varphi(X)$$

for any C^∞ vector fields X and Y on M.

$$(\nabla_X \varphi)Y = \beta\{g(\varphi(X),\ Y)\xi + \eta(Y)\varphi(X)\}$$

for any C^∞ vector fields X and Y on M and β is a nonzero constant on M. Using above formula, one can deduce for a β-Kenmotsu manifolds

$$\nabla_X \xi = \beta\{X + \eta(X)\xi\}$$

and

$$(\nabla_X \eta)(Y) = \beta\{g(X,\ Y) + \eta(X)\eta(Y)\}$$

Definition 2.1. A differentiable manifold M of dimension $(2n + 1)$ is called a δ-Lorentzian manifold if it admits a (1, 1) tensor field φ, a contravariant vector field ξ, a covariant vector field η and an indefinite metric g which satisfy

(2.1) $$\varphi^2 X = X + \eta(X)\xi,\ \eta(\xi) = -1,\ \eta(\varphi(X)) = 0$$

(2.2) $$g(\xi,\ \xi) = -\delta,\ \eta(X) = \delta g(X,\ \xi)$$

(2.3) $$g(\varphi(X), \varphi(Y)) = g(X, Y) + \delta\eta(X)\eta(Y),$$

where δ is such that $\delta^2 = 1$ and for any vector field X, Y on M. The structure defined above is called a δ-Lorentzian almost contact metric structureso. Manifold M togather with the structure $(\varphi, \xi, \eta, g, \delta)$ is called a δ-Lorentzian Kenmotsu manifold if

$$(\nabla_X \varphi)Y = g(\varphi(X), Y)\xi + \delta\eta(Y)\varphi(X)$$

Definition 2.2. A δ-Lorentzian almost contact metric manifold $M(\varphi, \xi, \eta, g, \delta)$ is called a ***δ-Lorentzian β-Kenmotsu manifold*** if

(2.4) $$(\nabla_X \varphi)(Y) = \beta\{g((\varphi(X), Y)\xi + \delta\eta(Y)\varphi(X)\},$$

where ∇ is the Levi-Civita connection with respect to g, β is a smooth function on M and X, Y are any vector fields on M and δ is such that $\delta^2 = 1$.

If $\delta = 1$, then δ-Lorentzian β-Kenmotsu manifold is the usual ***Lorentzian β-Kenmotsu manifold*** and is called the time like manifold. In this case, ξ is called a time like vector field.

Example 2.1. Let M be a δ-Lorentzian β-Kenmotsu manifold. We put

$$\overline{\varphi} = \varphi, \ \overline{\xi} = -\xi, \ \overline{\eta} = -\eta, \ \overline{g} = -g, \ \overline{\delta} = -\delta$$

Then $\left(\overline{\varphi}, \overline{\xi}, \overline{\eta}, \overline{g}, \overline{\delta}\right)$ defines a δ-Lorentzian almost contact structure on M. For,

$$\overline{\varphi}^2 X = X + \overline{\eta}(X)\xi, \ \overline{g}(\overline{\xi}, \overline{\xi}) = -\overline{\delta}, \ \overline{\eta}(\overline{\xi}) = -1, \ \overline{\eta}(X) = \overline{\delta}\overline{g}(X, \overline{\xi})$$

$$\overline{g}(\overline{\varphi}(X), \overline{\varphi}(Y) = \overline{g}(X, Y) + \overline{\delta}\overline{\eta}(X)\overline{\eta}(Y)$$

which by Definition 2.2, $\left(\overline{\varphi}, \overline{\xi}, \overline{\eta}, \overline{g}, \overline{\delta}\right)$ is a δ Lorentzian almost contact metric structure and further, it is a δ Lorentzian contact metric structure on M. Thus we conclude that if $(\varphi, \xi, \eta, g, \delta)$ is a δ-Lorentzian contact metric structure on M, then $\left(\overline{\varphi}, \overline{\xi}, \overline{\eta}, \overline{g}, \overline{\delta}\right)$ is also a δ Lorentzian contact metric structure on M.

Suppose $(\varphi, \xi, \eta, g, \delta)$ is a δ-Lorentzian normal contact metric structure on M. Since the parallelism with respect to g and $\overline{g}$ are the same, we get

$$(\overline{\nabla}_X\overline{\varphi})(Y) = (\nabla_X\varphi)(Y) = g(\varphi(X), Y)\xi + \delta\eta(Y)\varphi(X)$$

from which, finally we have

$$(\overline{\nabla}_X\overline{\varphi})(Y) = \overline{g}(\overline{\varphi}(X), Y)\overline{\xi} + \overline{\delta}\overline{\eta}(Y)\overline{\varphi}(X)$$

which shows that $(\overline{\varphi}, \overline{\xi}, \overline{\eta}, \overline{g}, \overline{\delta})$ is also a δ- Lorentzian normal contact metric structure on *M*. Similar arguments hold for δ-Lorentzian β-Kenmotsu manifolds.

Lemma 2.1. *For a δ-Lorentzian β-Kenmotsu manifolds, we have*

(2.5) $$\nabla_X\xi = \delta\beta\{X + \eta(X)\xi\}$$

for any vector field X on M.

Proof. From (2.4) of Definition 2.2, we have

$$\nabla_X(\varphi(Y) - \varphi(\nabla_X Y) = \beta\{g(\varphi(X), Y)\xi + \delta\eta(Y)\varphi(X)\}$$

Now taking $Y = \xi$ in the above equation and using (2.1), we get

$$-\varphi(\nabla_X Y) = -\beta\delta\varphi(X)$$

Applying φ on both sides of the above equation and using the fact that $(\nabla_X g)(\xi, \xi) = 0$ and (2.1), we get (2.5)

Example 2.2. Let us consider the 3-dimensional manifold $M^3 = \{(x, y, z) \in R^3\}$, where (x, y, z) are the standered co-ordinates in R^3. Following vector fields are linearly independent at each point of M^2.

$$e_1 = f(z)\frac{\partial}{\partial x} + g(z)\frac{\partial}{\partial y},\quad e_2 = -g(z)\frac{\partial}{\partial x} + f(z)\frac{\partial}{\partial y},\quad e_3 = \frac{\partial}{\partial z}$$

where f and g are given by

$$f = ae^{-\beta z}$$

$$g = be^{-\beta z}$$

with $f^2 + g^2 \neq 0$ for constants a and b and β. Let g be an indefinite metric defined by

$$g(e_1, e_2) = g(e_1, e_3) = g(e_2, e_3) = 0,\ g(e_e, e_1) = g(e_2, e_2) = 1,\ g(e_3, e_3) = -\delta$$

and the δ-Lorentzian metrc g is thus given by

$$g = \frac{1}{f^2 + g^2}\left\{(dx)^2 + (dy)^2 - \delta(dz)^2\right\}$$

where $\delta = \pm 1$. If $\delta = -1$, then δ-Lorentzian metric g becomes a Riemannian positive definite metric on M so that in this case, the characteristic vector field ξ becomes a space like and if $\delta = 1$, then it becomes a light like.

Let η be the 1-form defined by

$$\eta(X) = \delta g(X, \xi)$$

for any vector field on M^3. Let φ be the tensor field of type (1,1) defined by

$$\varphi(e_1) = -e_1,\ \varphi(e_2) = -e_2,\ \varphi(e_3) = 0$$

Using the linearity property of g and φ, one can deduce

$$\varphi^2 X = X + \eta(X)\xi,\ \eta(X) = -1,\ g(\xi, \xi) = -\delta,\ g(\varphi(X), \varphi(Y)) = g(X, Y) + \delta\eta(X)\eta(Y)$$

Also

$$\eta(e_1) = 0,\ \eta(e_2) = 0,\ \eta(e_3) = -1$$

for any vector field X and Y on M. Let ∇ be the Levi-Civita connection with respect to g. Then we have

$$[e_1, e_2] = 0,\ [e_1, e_3] = \delta\beta e_1,\ [e_2, e_3] = \delta\beta e_2$$

Using Koszule's formulas for Levi-Civita connection ∇ with respect to g, that is

$$2g(\nabla_X, Y, Z) = Xg(Y, Z) + Yg(Z, X) - Zg(X, Y)$$
$$- g(X, [Y, Z]) - g(Y, [X, Z]) + g(Z, [X, Y])$$

one can easily calculate

$$\nabla_{e_1} e_3 = \delta\beta_{e_1},\ \ \nabla_{e_3} e_3 = 0,\ \ \nabla_{e_2} e_3 = \delta\beta e_2$$

$$\nabla_{e_2} e_2 = \delta\beta_{e_3},\ \ \nabla_{e_1} e_2 = 0,\ \ \nabla_{e_2} e_1 = 0$$

$$\nabla_{e_1} e_1 = \delta\beta_{e_3}, \quad \nabla_{e_3} e_2 = \delta\beta e_2, \quad \nabla_{e_3} e_1 = \delta\beta e_1$$

With these information, the structure $(\eta, \xi, \eta, g, \delta)$ satisfies (2.4) and (2.5). Hence $M^3(\varphi, \xi, \eta, g, \delta)$ defines a δ-Lorentzian β-Kenmotsu manifold.

Lemma 2.2. *for a δ-Lorentzian β-Kenmotsu manifold M, we have*

(2.6) $$(\nabla_X \eta)(Y) = \beta\{g(X, Y) + \delta\eta(X)\eta(Y)\}$$

for any vector fields X and Y on M.

Proof. Consider,

(2.7) $$\begin{aligned}(\nabla_X \eta)(Y) &= \nabla_X(\eta(Y)) - \eta(\nabla_X Y) \\ &= \delta\nabla_X(g(Y, \xi)) - \delta g(\nabla_X Y, \xi) \\ &= \delta g(Y, \nabla_X \xi)\end{aligned}$$

Using (2.5) in (2.7) we get (2.6).

Lemma 2.3. *For a δ-Lorentzian β-Kenmotsu manifold M, we have*

(2.8) $$R(X, Y)\xi = \beta^2\{\eta(Y)X - \eta(X)Y\} + \delta\{(X\beta)\varphi^2 Y - (Y\beta)\varphi^2 X\}$$

(2.9) $$R(\xi, Y)\xi = \{\beta^2 + \delta(\xi\beta)\}\varphi^2 Y, \quad R(\xi, \xi)\xi = 0$$

for any vector fields X and Y on M.

Proof. From (2.5) and the fact that

$$[X, Y] = \nabla_X Y - \nabla_Y X$$

we have

(2.10) $$\begin{aligned}R(X, Y)\xi &= \nabla_X \nabla_Y \xi - \nabla_Y \nabla_X \xi - \nabla_{[X, Y]}\xi \\ &= \nabla_X\{\delta\beta\varphi^2 Y\} - \nabla_Y\{\delta\beta\varphi^2 X\} \\ &\quad -\delta\beta[\nabla_X Y - \nabla_Y X) + \eta(\nabla_X Y - \nabla_Y X)\xi] \\ &= \delta\{(X\beta)\varphi^2 Y - (Y\beta)\varphi^2 X\} + \beta\delta\{\nabla_X(\varphi^2 Y) - \nabla_Y(\varphi^2 X)\} \\ &\quad - \delta\beta[(\nabla_X Y - \nabla_Y X) + \eta(\nabla_X Y - \nabla_Y X)\xi],\end{aligned}$$

Also, we have

(2.11) $\nabla_X(\varphi^2 Y) = \nabla_X Y + \beta\{g(X, Y) + \delta\eta(X)\eta(Y)\}\xi + \eta(\nabla_X Y)\xi + \delta\beta\eta(Y)\{X + \eta(X)\xi\}$

From (2.11), finding the expression for

$$\nabla_X(\varphi^2 Y) - \nabla_Y(\varphi^2 X)$$

and further, substituting in (2.10), after simplification, we get (2.8).

(2.9) follows from (2.8) by putting $X = \xi$.

Lemma 2.4. *For a δ-Lorentzian β-Kenmotsu manifold M, we have*

(2.12) $$R(\xi, Y)X = \beta^2\{\delta g(X, Y)\xi - \eta(X)Y\} + \delta\{(X\beta)\varphi^2 Y - g(\varphi(X), \varphi(Y))(grad\beta)\}$$

for any vector fields X and Y on M.

Proof. From the identity

$$g(R(\xi, Y)X, Z) = g(R(X, Z)\xi, Y)$$

and (2.8) of Lemma 2.3, we have

$$g(R(\xi, Y)X, Z) = g(R(X, Z)\xi, Y)$$
$$= \beta^2\{-\eta(X)g(Z, Y) + \eta(Z)g(X, Y)\}$$
$$+ \delta\{-(Z\beta)g(\varphi^2 X, Y) + (X\beta)g(Z, \varphi^2 Y)\}$$

After simplification, we (2.12).

Lemma 2.5. *For a δ-Lorentzian β-Kenmotsu manifold M, we have*

(2.13) $$S(Y, \xi) = 2n\beta^2\eta(Y) - (2n - 1)\delta(Y\beta) + \delta\eta(Y)(\xi\beta)$$

(2.14) $$S(\xi, \xi) = -2n\{\beta^2 + \delta(\xi\beta)\}$$

(2.15) $$QY = 2n\beta^2 Y,$$

where β = *constant*

Proof. From (2.8), we have

(2.16) $$g(R(X, Y)\xi, Z) = \beta^2\{\eta(Y)g(X, Z) - \eta(X)g(Y, Z)\} + \delta\{(X\beta)g(\varphi^2 Y, Z) - (Y\beta)g(\varphi^2 X, Z)\}$$

Let $\{e_i\}$, $i = 1, 2, 3..., 2n + 1$ be the orthonormal basis at each point of the tangent space of M. Then Putting $X = Z = e_i$, we have

$$g(R(e_i, Y)\xi, e_i) = \beta^2\{\eta(Y)g(e_i, e_i) - \eta(e_i)g(Y, e_i)\} + \delta\{(e_i\beta)g(\varphi^2 Y, e_i) - (Y\beta)g(\varphi^2 e_i, e_i)\}$$

which after simplification gives (2.13). Put $Y = \xi$ in (2.13) to get (2.14). Also from (2.13), we get (2.15).

3. GENERALISED RECURRENT δ-LORENTZIAN β-KENMOTSU MANIFOLD

In this Section onwards, we assume that β is constant.

Definition 3.1. A δ-Lorentzian β-Kenmotsu manifold M is said to be a generalised recurrent manifold if the curvature tensor R of M satisfies

(3.1) $$(\nabla_X R)(Y, Z)W = A(X)R(Y, Z)W + B(X)\{g(Z, W)g(Z, W)Y - g(Z, W)Z\},$$

where A and B are associated 1-forms and X, Y, Z, W are any vector fields on M.

Lemma 3.1. *For a generalised recurrent* δ*-Lorentzian* β*-Kenmotsu manifold* M*, we have*

(3.2) $$(\nabla_X R)(\xi, Z)\xi = 0$$

for any vector fields X, Z *on* M.

Proof. We know that

$$\begin{aligned}(\nabla_X R)(\xi, Z)\xi &= \nabla_X(R(\xi, Z)\xi - R(\nabla_X\xi, Z)\xi \\ &\quad - R(\xi, \nabla_X Z)\xi - R(\xi, Z)\nabla_X\xi \\ &= \nabla_X[\{\beta^2 + \delta(\xi\beta)\}\varphi^2 Z] \\ &\quad - R[\delta\beta\{X + \eta(X)\xi\}, Z]\xi \\ &\quad - \{\beta^2 + \delta(\xi\beta)\}\varphi^2(\nabla_X Z) \\ &\quad - \delta\beta R(\xi, Z)X - \delta\beta\eta(X)R(\xi, Z)\xi\end{aligned}$$

Now using (2.5), (2.6), (2.8), (2.9) in the above equation, after lengthy simplification, we get (3.2)

We assume that M is a generalised recurrent δ-Lorentzian β-Kenmotsu manifold. Then (3.1) of Definition 3.1 holds. Now put $Y = W = \xi$ in (3.1), we find

(3.3) $$(\nabla_X R)(\xi, Z)\xi = A(X)R(\xi, Z)\xi - B(X)\{g(Z, \xi)\xi - g(\xi, \xi)Z\}$$

By virtue of (3.2) of Lemma 3.1 and the above equation (3.3), we find

$$\beta^2 A(X) + \delta B(X) = 0$$

for any vector field X on M. Hence we state

Theorem 3.1. *A generalised recurrent δ- Lorentzian β-Kenmotsu manifold M satisfies*

$$\beta^2 A + \delta B = 0$$

where $\delta = \pm 1$.

Corollary 3.1. *A generalised recurrent Lorentzian β-Kenmotsu manifold M satisfies* $\beta^2 A + B = 0$

Corollary 3.2. *A generalised recurrent Lorentzian Kenmotsu manifold M the 1-forms A and B are in the opposite direction.*

4. RICCI SYMMETRIC AND SEMISYMMETRIC δ-LORENTZIAN β-KENMOTSU MANIFOLD

In this Section, we introduce the notion of Ricci semisymmetric and semi symmetric δ-Lorentzian β-Kenmotsu manifold

Definition 4.1. A δ-Lorentzian β-Kenmotsu manifold is said to be Ricci semisymmetric if

(4.1) $$(R(X, Y)\cdot S)(Z, U) = 0$$

and is said to be semisymmetric if

(4.2) $$(R(X, Y)\cdot R)(Z, U) = 0$$

for any vector fields X, Y, Z, U on M.

We have

(4.3) $(R(X, Y)\cdot S)(Z, U) = R(X, Y)S(Z, U) - S(R(X, Y)Z, U) - S(Z, R(X, Y)U)$

From (4.1) the above equation (4.3) reduces to

(4.4) $S(R(X, Y)Z, U) + S(Z, R(X, Y)U) = 0$

Now setting $X = Z = \xi$ in (4.4), we get

(4.5) $S(R(\xi, Y)\xi, U) + S(\xi, R(\xi, Y)U) = 0$

Now using (2.12) of Lemma 2.4 and (2.9) of Lemma 2.3 in (4.5), we get

(4.6) $S(Y, U) = (-2n\beta^2\delta)g(Y, U)$

(4.7) $r = -2n(2n + 1)\beta^2\delta$

where r is the scalar curvature of M. Hence, we state

Theorem 4.1. *A Ricci symmetric δ-Lorentzian β-Kenmotsu manifold is an Einstein manifold*

Corollary 4.1. [9] *A Ricci symmetric Lorentzian β-Kenmotsu manifold is an Einstein manifold*

Corollary 4.2. [9] *A Ricci symmetric Lorentzian Kenmotsu manifold is an Einstein manifold*

Theorem 4.2. *A symmetric δ-Lorentzian β-Kenmotsu manifold is an Einstein manifold*

Proof. Follows from the fact that R·R = 0 is the subset of R·S = 0, so that R·S = 0 implies that R·R = 0. Hence (4.4) holds.

Corollary 4.3. [9] *A symmetric Lorentzian β-Kenmotsu manifold is an Einstein manifold*

Theorem 4.3. *For a Ricci semisymmetric δ-Lorentzian β-Kenmotsu manifold M, the scalar curvature r of M is constant and is given by (4.7)*

5. A δ-LORENTZIAN β-KENMOTSU MANIFOLD WITH C=0

The Weyl's conformal curvature tensor C of type (1, 3) on M is defined by (5.1)

$$C(X, Y)Z = R(X, Y)Z + \frac{1}{2n+1}[S(X, Z)Y - S(Y, Z)X + g(X, Z)QY - g(X, Z)QX]$$

$$+ \frac{1}{2n(2n+1)}\{g(X, Z)Y - g(Y, Z)X\},$$

where $S(X, Y) = g(QX, Y)$ and X, Y, Z are any vector fields on M.

For $n > 1$ it is well known that M is conformally flat if C is identically vanishes on M.

Theorem 5.1. *A conformally flat δ-Lorentzian β-Kenmotsu manifolds $M(n > 1)$ is an η-Einstein manifold.*

Proof. Suppose M is conformally flat. Then $C \equiv 0$, so that (5.1) takes the form

$$(5.2) \qquad R(X, Y)Z = -\frac{1}{2n+1}[S(X, Z)Y - S(Y, Z)X + g(X, Z)QY - g(X, Z)QX]$$
$$-\frac{1}{2n(2n+1)}\{g(X, Z)Y - g(Y, Z)X\},$$

Set $Z = \xi$ in (5.2) and then using (2.13) of Lemma 2.5 and (2.8) of Lemma 2.3, one obtains

$$\eta(X)QY - \eta(Y)QX = 2n\beta^2\{\eta(Y)X - \eta(X)Y\}$$
$$- (2n - 1)\beta^2\{\eta(Y)X - \eta(X)Y\}$$
$$-\frac{\delta r}{2n}\{-\eta(X)Y + \eta(Y)X\}$$

which after simplification gives

$$(5.3) \qquad \eta(X)QY - \eta(Y)QX = \left\{\beta^2 - \frac{\delta r}{2n}\right\}\{\eta(Y)X - \eta(X)Y\}$$

Putting $Y = \xi$ in (5.3) and using (2.15) of Lemma 2.5, we have

$$QX = \left\{\frac{\delta r}{2n} - \beta^2\right\}X - \left\{(2n-1)\beta^2 - \frac{\delta r}{2n}\right\}\eta(X)\xi.$$

From which, we have

$$(5.4) \qquad S(X, Y) = \left\{\frac{\delta r}{2n} - \beta^2\right\}g(X,Y) - \left\{(2n-1)\beta^2 - \frac{\delta r}{2n}\right\}\eta(X)\eta(Y),$$

which proves the Theorem.

Contracting (5.4), we have the following expression for the scalar curvature r of M

(5.5) $$r = \frac{[2n(2n+1)\delta - 1]\beta^2}{2n - (2n+1)\delta + 1}$$

provided $\delta \neq 1$. If $\delta = -1$, then (5.5), we have

(5.6) $$r = \frac{2n(n+1)\beta^2}{2n+1}$$

Theorem 5.2. *In a conformaly flat δ-Lorentzian β-Kenmotsu manifolds M* ($n > 1$), *the scalar curvature r of M is given by* (5.5)

Theorem 5.3. *In a conformally flat δ-Lorentzian β-Kenmotsu manifolds M* ($n > 1$), *the scalar curvature r of M is constant and given by* (5.6).

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