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SOMEWHAT CONTINUOUS FUNCTIONS ON FUZZIFIED TOPOLOGICAL SPACE

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ABSTRACT : In this paper the concept of somewhat continuous functions, somewhat open functions are introduced in fuzzified topological space and some interesting properties of these functions are investigated.

Key words : Fuzzified topological space, somewhat continuous functions, somewhat open function.
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1. INTRODUCTION

The theory of fuzzy topological spaces was introduced and developed by C.L. Chang [2] and since then various notions in classical topology have been extended to fuzzy topological spaces. The concept of somewhat functions was introduced by Karl R. Genry and Hughes B. Hoyle [3] and this concept was studied in connection with the idea of feebly continuous function and feebly open functions introduced by Zdenek Frolik [4]. In [5] G. Thangaraj and G. Balasubranian introduced these concepts in fuzzy topological spaces. In this paper we have discussed the properties of somewhat continuous and somewhat open functions in fuzzified topological spaces.

2. PRELIMINARIES

In this section we recall some definitions and results that will be used in the sequel.

Definition 2.1[7] A fuzzified topology on a nonempty set X is fuzzy subset of power set of X, P(X) i.e., a function $\tau : P(X) \rightarrow [0, 1]$, satisfying the following conditions:

$\tau(X) = \tau(\phi) = 1$

$\tau(A \cap B) \geq \tau(A) \wedge \tau(B)$, A, B $\in$ P(X)

$\tau(\cup A_i) \geq \wedge_i \tau(A_i)$ for any sub collection $\{A_i\}$ of P(X).

The pair (X, τ) will be called a fuzzified topological space (FTS). $\tau(A)$ is the degree of openness of A and $\tau(A^c)$ is the degree of closedness of A, where A^c is the complement of A.

Definition 2.2[7] A fuzzified co-topology on a nonempty set X is a fuzzy subset of P(X) i.e., a function $\omega : P(X) \to [0, 1]$, satisfying the following conditions:

$\omega(X) = \omega(\phi) = 1$

$\omega(A \cup B) \geq \omega(A) \wedge \omega(B)$, A, B$\in$ P(X)

$\tau(\cap A_i) \geq \wedge_i \tau(A_i)$ for any sub collection $\{A_i\}$ of P(X).

Proposition 2.3[7] Let τ be a fuzzified topology on X and $\omega_\tau : X \to [0, 1]$ be defined as $\omega_\tau(A) = \tau(A^c)$. Then ω_τ is a fuzzified co-topology on X.

Proposition 2.4[7] Let ω be a fuzzified co-topology on X and $\tau_\omega : X \to [0, 1]$ be defined as $\tau_\omega(A) = \omega(A^c)$. Then τ_ω is a fuzzified topology on X.

Proposition 2.5[7] If τ is a fuzzified topology and ω is a fuzzified co-topology on X then $\tau_{\omega\tau} = \tau$ and $\omega_{\tau\omega} = \omega$

Proposition 2.6[7] Let (X, τ) be a fuzzified topology and $Y \subseteq X$. Then $\tau_Y : P(Y) \to [0, 1]$ given by

$\tau_Y(U) = \vee\{\tau(V) : U = Y \cap V\}$ is a fuzified topology on Y.

τ_Y is then called fuzzified subspace topology.

Definition 2.7[7] Let (X, τ) and (Y, δ) be two FTSs. A function $f : (X, \tau) \to (Y, \delta)$ is said to be continuous with respect to τ and δ if $\tau(f^{-1}(U)) \geq \delta(U)$ for each $U \in P(Y)$.

Result 2.8. Let (X, τ) and (Y, δ) be two FTSs. A function $f : (X, \tau) \to (Y, \delta)$ is continuous with respect to τ and δ iff $\omega_\tau(f^{-1}(U)) \geq \omega_\delta(U)$ for each $U \subseteq Y$.

Definition 2.9[7] Let (X, τ) and (Y, δ) be two FTSs. A function $f : (X, \tau) \to (Y, \delta)$ is said to be open with respect to τ and δ if $\tau(G) \leq \delta(f(G))$ for each $G \in P(X)$.

Definition 2.10[7] Let (X, τ) and (Y, δ) be two FTSs. A function $f : (X, \tau) \to (Y, \delta)$ is said to be closed with respect to τ and δ if $\omega_\tau(G) \leq \omega_\delta(f(G))$ for each $G \in P(X)$.

3. SOMEWHAT CONTINUOUS FUNCTIONS IN FUZZIIFIED TOPOLOGICAL SPACE

In this section we introduce and discuss the properties of somewhat continuous function in fuzzified topological spaces.

Definition 3.1. Let (X, τ) and (Y, δ) be two FTSs. A function $f : (X, \tau) \to (Y, \delta)$ is said to be somewhat continuous with respect to τ and δ if for each $U \subseteq Y$, with $f^{-1}(U) \neq \phi$ there exist $\phi \neq V \subseteq X$ such that $V \subseteq f^{-1}(U)$ and $\tau(V) \geq \delta(U)$.

It is clear from the definition that a continuous function is somewhat continuous. That the converse is not always true is evident from the following example.

Example 3.2. Let X = {a, b, c}, μ and ν are fuzzified topologies on X defined as :

	{a}	{b}	{c}	{a, b}	{a, c}	{b, c}	ϕ	X
μ	.7	.5	.1	.5	.3	.1	1	1
ν	.1	0	.1	.6	.1	0	1	1

Let $g : (X, \mu) \to (Y, \nu)$ be the identity function. Then $\mu[g^{-1}(\{a, b\})] = \mu(\{a, b\} = .5 < .6 = \nu(\{a, b\})$. Therefore g is not continuous. However there exists $\{a\} \subseteq \{a, b\}$ such that $\mu(\{a\}) > \nu(\{a, b\})$. Hence g is somewhat continuous.

Result 3.3. A function $f : (X, \tau) \to (Y, \delta)$ is somewhat continuous with respect to τ and δ iff for each $U \subseteq Y$, with $f^{-1}(U) \neq X$ there exist $V \subsetneq X$ such that and $f^{-1}(U) \subseteq V$ and $\omega_\tau(V) \geq \omega_\delta(U)$.

Proof. Let $f : (X, \tau) \to (Y, \delta)$ be somewhat continuous. Consider $U \subseteq Y$ such that $f^{-1}(U) \neq X$. Then $U^c \subseteq Y$ such that $f^{-1}(U^c) \neq \phi$.

As f is somewhat continuous, there exists $\phi \neq W \subseteq X$:

$W \subseteq f^{-1}(U^c)$ and $\tau(W) \geq \delta(U^c) \Rightarrow W \subseteq \{f^{-1}(U)\}^c$ and $\omega_\tau(W^c) \geq \omega_\delta(U)$

$\Rightarrow f^{-1}(U) \subseteq W^c$ and $\omega_\tau(W^c) \geq \omega_\delta(U)$.

Let $V = W^c$, then $V \subsetneq X : f^{-1}(U) \subseteq V$ and $\omega_\tau(V) \geq \omega_\delta(U)$

Converse.

Let $U \subseteq Y$ such that $f^{-1}(U) \subseteq X$. Then $U^c \subseteq Y$ such that $f^{-1}(U^c) \neq \phi$. So by hypothesis there exist $V \subsetneq X : f^{-1}(U^c) \subseteq V$ and $\omega_\tau(V) \geq \omega_\delta(U^c)$ which gives $\{f^{-1}(U)\}^c \subseteq V$ and $\omega_\tau(V) \geq \omega_\delta(U^c) \Rightarrow V^c \subseteq f^{-1}(U)$ and $\tau(V^c) \geq \delta(U)$. Let $V^c = W$.

Then $W \neq \phi$ and $W \subseteq f^{-1}(U)$ and $\tau(W) \geq \delta(U)$.

Result 3.4. Let (X, τ), (Y, δ) and (Z, η) be FTSs.

Let $f : (X, \tau) \to (Y, \delta)$ and $g : (Y, \delta) \to (Z, \eta)$ are somewhat continuous funcitons, then $g o f : (X, \tau) \to (Z, \eta)$ is continuous, if $f^{-1}(V) \neq \phi$ for all $\phi \neq V \subseteq Y$.

Proof. Let $U \subseteq Z$ such the $(gof)^{-1}(U) \neq \phi$.

We have $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U) \neq \phi$ implies $g^{-1}(U) \neq \phi$. As g is somewhat continuous, there exists $\phi \neq W \subseteq Y : W \subseteq g^{-1}(U)$ and $\delta(W) \geq \eta(U)$.

Again by hypothesis $\phi \neq W \Rightarrow f^{1}(W) \neq \phi$. As f is somewhat continuous, there exists $\phi \neq V \subseteq X : V \subseteq f^{-1}(W)$ and $\tau(V) \geq \delta(W)$. Thus $V \subseteq f^{-1}(g^{-1}(U))$ and $\tau(V) \geq \delta(W) \geq \eta(U)$.

That is, there exists $\phi \neq V \subseteq X : V \subseteq (gof)^{-1}(U))$ and $\tau(V) \geq \eta(U)$.

The following example shows that the condition $f^{-1}(V) \neq \phi$ for all $\phi \neq V \subseteq Y$ is sufficient but not necessary.

Example 3.5. Let X = {a, b, c}, τ, μ and ν are fuzzified topologies on X defined as :

	{a}	{b}	{c}	{a, b}	{a, c}	{b, c}	φ	X
μ	.7	.5	.1	.5	.3	.1	1	1
ν	.1	0	.0	.6	.1	0	1	1
τ	.7	.4	0	.4	0	.1	1	1

Let $f : (X, \mu) \to (X, \tau)$ be defined as $f(a) = a$, $f(b) = f(c) = c$ and $g : (X, \tau) \to (X, \nu)$ be the identity function.

Then it is easy to verify that f, g and *gof* are somewhat continuous though $f^{-1}(\{b\}) = \phi$.

Definition 3.6. Let (X, τ) be a FTS. A subset $D \subseteq X$ is said to be dense in X if there exists no subset $V \subsetneq X$ such that $D \subseteq V$ and $\omega_\tau(V) > \omega_\tau(D)$.

Result 3.7. Image of a dense subset under a somewhat continuous and closed function is dense.

Proof. $f : (X, \tau) \to (Y, \mu)$ be somewhat continuous and D be dense in X. Consider $f(D)$. If $f(D) = X$, we are done. So let $f(D) \neq X$. Suppose $f(D)$ is not dense, then there exists $B \subsetneq Y : f(D) \subsetneq B$ and $\omega_\mu(B) > \omega_\mu(f(D))$. Since f is somewhat continuous, for $B \subsetneq Y$ there exists $F \subsetneq X : f^{-1}(B) \subseteq F$ and $\omega_\tau(F) \geq \omega_\mu(B)$. Again $f(D) \subseteq B \Rightarrow D \subseteq f^{-1}(B) \subseteq F \Rightarrow D \subseteq F$. Also, $\omega_\tau(F) \geq \omega_\mu(B) > \omega_\mu(f(D)) \geq \omega_\tau(D)$, since f is closed. But this is a contradiction to the fact that D is dense in X.

The following example shows that the above result need not be true if the condition of closed of f is omitted.

Example 3.8. Let X = {a, b, c, d} and τ and μ be fuzzified topologies on X defined as:

$\tau(A) = 1,\ A = X, \phi$

$= .7,\ A = \{c, d\}$

$= .5$, otherwise.

$\mu(A) = 1,\ A = X, \phi$

$= .6,\ A = \{b\}$

$= .4$, otherwise.

Let $f : (X, \tau) \to (Y, \mu)$ be given by $f(a) = b$, $f(b) = c$, $f(c) = d$, $f(d) = a$.

Then f is somewhat continuous and $D = \{a, b\}$ is dense in X.

But $f(D) = \{b, c\}$ is not dense in Y, as there exists $\{a, c, d\} \supseteq \{b, c\}$ such that $\omega_\mu(\{a, c, d\}) = \mu(\{b\}) = .6 > .4 = \mu(\{a, d\}) = \omega_\mu(\{b, c\}) = \omega_\mu(f(D))$.

Note that here f is not closed.

Result 3.9. Let $f : (X, \tau) \to (Y, \delta)$ be somewhat continuous. Then for any $A \subseteq X$,

$f/_A : (A, \tau_A) \to (Y, \delta)$ is somewhat continuous.

Proof. Since $f/_A$ is the composition of the inclusion map $j : (A, \tau_A) \to (X, \tau)$ and $f: (X, \tau) \to (Y, \delta)$, the result follows from Result 3.4.

Result 3.10. Let $f : (X, \tau) \to (Y, \delta)$ be somewhat continuous where Y is a subspace of Z. Then $h : X \to Z$, where h is obtained by expanding the range of f is somewhat continuous.

Proof. Let μ denote the fuzzified topology on Z. Then $\delta = \mu_Y$. Let $U \subseteq Z$ with $h^{-1}(U) \neq \phi$. We have $V = Y \cap U \subseteq U$. Then $f^{-1}(V) = h^{-1}(V) = h^{-1}(Y \cap U) = h^{-1}(Y) \cap h^{-1}(U) = h^{-1}(U)$. So $f^{-1}(V) \neq \phi$. Since f is somewhat continuous, there exists $\phi \neq W \subseteq X : W \subseteq f^{-1}(V)$ and $\tau(W) \geq \delta(V) = \mu_Y(V) \geq \mu(V)$. Thus $W \subseteq h^{-1}(U)$ and $\tau(W) \geq \mu(V)$.

Result 3.11. Let $f : (X, \tau) \to (Y, \delta)$ be somewhat continuous and $f(X) \subseteq Z \subseteq Y$.

Then $g : (X, \tau) \to (Z, \delta_z)$ is somewhat continuous.

Proof. Let $U \subseteq Z$ with $g^{-1}(U) \neq \phi$.

As $f(X) \subseteq Z$, then for any $V \subseteq Y : U = Z \cap V$, $f^{-1}(V) = g^{-1}(U)$.

As f is somewhat continuous, there exists $\phi \neq W \subseteq X : W \subseteq f^{-1}(V)$ and $\tau(W) \geq \delta(V)$.

But this is true for any $V \subseteq Y$ satisfying $U = Z \cap V$.

So $\tau(W) \geq \wedge\{\delta(V) : U = Z \cap V\} = \delta_z(U)$.

Hence there exists $\phi \neq W \subseteq X : W \subseteq g^{-1}(V)$ and $\tau(W) \geq \delta_z(U)$.

Result 3.12. Let $f : (X, \tau) \to (Y, \delta)$ and suppose A and B be subsets of X :

$X = A \cup B$ and $\tau(A) = 1 = \tau(B)$. If $f_A : (A, \tau_A) \to (Y, \delta)$ and $f_B : (B, \tau_B) \to (Y, \delta)$ are somewhat continuous then f is somewhat continuous.

Proof. Let $U \subseteq Y$ and $f^{-1}(U) \neq \phi$. As $f^{-1}(U) = f_A^{-1}(U) \cup f_B^{-1}(U)$, either $f_A^{-1}(U) \neq \phi$ or $f_B^{-1}(U) \neq \phi$. Suppose $f_A^{-1}(U) \neq \phi$. As f_A is somewhat continuous, there exists $\phi \neq W \subseteq A : W \subseteq f_A^{-1}(U)$ and $\tau_A(W) \geq \delta(U)$. But $\tau(A) = 1$ implies $\tau_A(W) = \tau(W)$.

Thus we have $W \subseteq f_A^{-1}(U) \subseteq f^{-1}(U)$ and $\tau(W) \geq \delta(U)$.

Definition 3.13. Let τ and δ be two fuzzified topologies on a set X. τ is said to be weakly equivalent to δ if for any $\phi \neq W \subseteq X$, there exists $\phi \neq U, V \subseteq W$ such that $\tau(U) \geq \delta(W)$ and $\delta(V) \geq \tau(W)$.

Remark 3.14. τ and δ are weakly equivalent on X iff the identity function from (X, τ) onto (X, δ) is somewhat continuous in both directions.

Proof. The proof is straightforward.

Result 3.15. Let $f : (X, \tau) \to (Y, \delta)$ be somewhat continuous. If μ is weakly equivalent to τ, then $f : (X, \mu) \to (Y, \delta)$ is somewhat continuous.

Proof. Let $U \subseteq Y$ and $f^{-1}(U) \neq \phi$. As f is somewhat continuous, there exists $\phi \neq V \subseteq X$: $V \subseteq f^{-1}(U)$ and $\tau(V) \geq \delta(U)$.

Again μ is weakly equivalent to δ, therefore there exists $\phi \neq W \subseteq V : \mu(W) \geq \tau(V)$.

Result 3.16. Let $f : (X, \tau) \to (Y, \delta)$ be somewhat continuous and onto. If δ is weakly equivalent to ν, then $f : (X, \tau) \to (Y, \nu)$ is somewhat continuous.

Proof. Let $U \subseteq Y$ such that $f^{-1}(U) \neq \phi$. Then $U \neq \phi$. Again δ is weakly equivalent to ν implies there exists $\phi \neq W \subseteq U : \delta(W) \geq \nu(U)$. As f is onto $W \neq \phi$ implies $f^{-1}(W) \neq \phi$.

Now $f : (X, \tau) \to (Y, \delta)$ is somewhat continuous, therefore, there exists $\phi \neq V \subseteq X$ such that $V \subseteq f^{-1}(W)$ and $\tau(V) \geq \delta(W)$.

Thus there exists $\phi \neq V \subseteq X$ such that $V \subseteq f^{-1}(W)$ and $\tau(V) \geq \nu(U)$.

Combining Result 3.15 and 3.16 we have

Result 3.17. Let $f : (X, \tau) \to (Y, \delta)$ be somewhat continuous and onto. If μ is weakly equivalent to τ and ν is weakly equivalent to δ, then $f : (X, \mu) \to (Y, \nu)$ is somewhat continuous.

Definition 3.18. A fuzzified topological space is said to be separable if there exist a countable dense subset.

Result 3.19. Let $f : (X, \tau) \to (Y, \delta)$ be closed and somewhat continuous. If X is separable then so is Y.

Proof. Since image of a countable set is countable, the result follows from Result 3.7.

4. SOMEWHAT OPEN FUNCTIONS IN FUZZIFIED TOPOLOGICAL SPACE

In this section we introduce and discuss the properties of somewhat open function in fuzzified topological spaces.

Definition 4.1. Let (X, τ) and (Y, δ) be two FTSs. A function $f : (X, \tau) \to (Y, \delta)$ is said to be somewhat open with respect to τ and δ if for each $\phi \neq U \subseteq X$, there exist $\phi \neq V \subseteq X : V \subseteq f(U)$ and $\delta(V) \geq \tau(U)$.

It is clear from the definition that an open function is somewhat open. That the converse is not always true is evident from the following example.

Example 4.2. Let X = {a, b, c}, τ and μ are fuzzified topologies on X defined as :

	{a}	{b}	{c}	{a, b}	{a, c}	{b, c}	φ	X
τ	.1	.1	.1	.5	.1	.1	1	1
μ	.6	.2	.2	.2	.2	.2	1	1

Let $f : (X, \tau) \to (Y, \mu)$ be the identity function.

Then f is not open, since $f(\{a, b\}) = \{a, b\}$ and $\mu[f(\{a, b\})] = .2 < .5 = \tau\{a, b\}$.

However f is somewhat open as there exists $\{a\} \subseteq f(\{a, b\})$ such that

$\mu(\{a\}) = .6 > .5 = \tau\{a, b\}$

Definition 4.3. Let (X, τ) and (Y, δ) be two FTSs. A function $f : (X, \tau) \to (Y, \delta)$ is said to be somewhat closed with respect to τ and δ if for each $U \not\subseteq X$, there exist $V \not\subseteq Y$ such that $f(U) \subseteq V$ and $\omega_\delta(V) \geq \omega_\tau(U)$.

Result 4.4. Let $f : (X, \tau) \to (Y, \mu)$ is somewhat continuous, and $g : (Y, \mu) \to (Z, \nu)$ is somewhat continuous, the $(gof) : (X, \tau) \to (Z, \nu)$.

Proof. Let $\phi \neq U \subseteq X$. Consider $(gof)(U)$. Since $f : (X, \tau) \to (Y, \mu)$ is somewhat continuous, there exist $\phi \neq V \subseteq Y : V \subseteq f(U)$ and $\mu(V) \geq \tau(U)$. Again $g : (Y, \mu) \to (Z, \nu)$ is somewhat continuous, so there exists $\phi \neq W \subseteq Z : W \subseteq g(V)$ and $\nu(W) \geq \mu(V)$.

Thus we have $\phi \neq W \subseteq g(V) \subseteq g(f(U)) = (gof)(U)$ such that $\nu(W) \geq \tau(U)$.

Result 4.5. Inverse image of a dense subset under a somewhat closed and continuous function is dense.

Proof. Let $f : (X, \tau) \to (Y, \delta)$ be somewhat closed and continuous function. Let D be dense in Y. consider $f^{-1}(D)$. Suppose $f^{-1}(D)$ is not dense in X. Then there exists $F \not\subseteq X : f^{-1}(D) \subseteq F$ and $\omega_\tau(F) > \omega_\tau(f^{-1}(D))$. Now $F \not\subseteq X \Rightarrow f(F) \not\subseteq Y$. As f is somewhat closed, there exists $B \not\subseteq Y : f(F) \subseteq B$ and $\omega_\delta(B) \geq \omega_\tau(F)$.

Thus $D \subseteq f(F) \subseteq B$ and $\omega_\delta(B) \geq \omega_\tau(F) > \omega_\tau(f^{-1}(D)) \geq \omega_\delta(D)$, since f is continuous. But this is contradiction to that fact that D is dense in Y. Hence $f^{-1}(D)$ is dense in X.

Result 4.6. Let $f : (X, \tau) \to (Y, \delta)$ be one-one and onto. Then f is somewhat open iff f is somewhat colsed.

Proof. Suppose f is open. Let $U \subseteq X : f(U) \neq Y$. To find $V \not\subseteq Y$ such that $f(U) \subseteq V$ and $\omega_\delta(V) \geq \omega_\tau(U)$. If $U = \phi$, then $f(U) = \phi$. We choose $V = \phi$.

So let $\phi \neq U \not\subseteq X$. Then $\phi \neq f(U) \not\subseteq Y$. Let $U^c = W$, then $\phi \neq W \not\subseteq X$. Since f is somewhat open, there exists $\phi \neq V \subseteq Y$:

$V \subseteq f(W)$ and $\tau(W) \geq \delta(V) \Rightarrow f(W)^c \subseteq V^c$ and $\omega_\tau(W^c) \geq \omega_\delta(V^c)$

$\Rightarrow f(W^c) \subseteq V^c$ and $\omega_\tau(U) \geq \omega_\delta(V^c)$. Let $V^c = F$. Since $\phi \neq V$, $Y \neq V^c = F$. Thus for $U \subseteq X : f(U) \not\subseteq Y$, there exists $F \not\subseteq Y : f(U) \subseteq F$ and $\omega_\tau(U) \geq \omega_\delta(F)$. Hence f is somewhat closed.

Converse. Let $\phi \neq U \subseteq X$. If $U = X$, $f(U) = Y$. Choosing $V = Y$, we are done.

So let $\phi \neq U \not\subseteq X$. Let $G = U^c$, then $\phi \neq G \not\subseteq X$ and so $f(G) \not\subseteq Y$, as f is onto.

Hence by hypothesis there exists $W \not\subseteq Y$:

$f(G) \subseteq W$ and $\omega_\delta(W) \geq \omega_\tau(G) \Rightarrow W^c \subseteq f(G)^c$ and $\delta(W^c) \geq \tau(G^c)$

$\Rightarrow W^c \subseteq f(G^c)$ and $\delta(W^c) \geq \tau(G^c) \Rightarrow W^c \subseteq f(U)$ and $\delta(W^c) \geq \tau(U)$

Let $W^c = V$. Then $\phi \neq V \subseteq Y : V \subseteq f(U)$ and $\delta(V) \geq \tau(U)$. Hence f is somewhat open.

Result 4.7. Let $f : (X, \tau) \to (Y, \delta)$ and suppose A and B be subsets of X:

$X = A \cup B$. If $f : (A, \tau_A) \to (Y, \delta)$ and $f : (B, \tau_B) \to (Y, \delta)$ are somewhat open then f is somewhat open.

Proof. Let $\phi \neq U \subseteq X$. If $U \cap B = \phi$, then $f(U) = f_A(U)$.

Since f_A is somewhat open, there exists $\phi \neq V \subseteq Y : V \subseteq f_A(U)$ and $\delta(V) \geq \tau_A(U)$. But $\tau_A(U) \geq \tau(U)$. Therefore $\phi \neq V \subseteq Y : V \subseteq f(U)$ and $\delta(V) \geq \tau(U)$.

Similar is the case if $U \cap A = \phi$. So let $U \cap A \neq \phi \neq U \cap B$. Then $f(U) = f_A(U) \cup f_B(U)$.

As f_A is somewhat open, there existss $\phi \neq V \subseteq Y : V \subseteq f_A(U)$ and $\delta(V) \geq \tau_A(U) \geq \tau(U)$.

As f_B is somewhat open, there exists $\phi \neq W \subseteq Y : W \subseteq f_B(U)$ and $\delta(W) \geq \tau_A(U) \geq \tau(U)$.

Then there exists $\phi \neq F = V \cap W \subseteq Y$:

$F \subseteq f_A(U) \cup f_B(U) = f(U)$ and $\delta(F) = \delta(V \cap W) \geq \delta(V) \wedge \delta(W) \geq \tau(U)$.

Remark 4.8. τ and δ are weakly equivalent on X iff the identity function from (X, τ) onto (X, δ) is somewhat open in both directions.

Result 4.9. Let $f : (X, \tau) \to (Y, \delta)$ be somewhat open funciton. If μ is weakly equivalent to τ and ν is weakly equivalent to τ, then $f : (X, \mu) \to (Y, \nu)$ is somewhat open function.

Proof. The proof is straightforward.

Definition 4.10. A $f : (X, \tau) \to (Y, \delta)$ is a said to be somewhat homeomorphism if f is one-one, onto, somewhat open and somewhat continuous.

Remark 4.11. If f is a somewhat homeomorphism then f^{-1} is also a somewhat homeomorphism.

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