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STRONGLY θ-β-CONTINUOUS FUNCTIONS

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ABSTRACT : In the paper, we introduce a new class of functions called strongly *θ-β* continuous functions which is stronger than *β*-continuous functions and investigate their properties.

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1. INTRODUCTION

A subset A of topological space X is called *β*-open [1] or semi-preopen [3] if $A \subset Cl(\text{Int}(\text{Cl}(A)))$. A function $f : X \to Y$ is called *β*-continuous [1] if the preimage $f^{-1}(V)$ of each open set V of Y is *β*-open in X. Borsik and Doboš [5] introduced the notion of almost quasicontinuous functions to obtain decompositions of quasi continuity. Borsik [4], Ewert [6], and Popa and Noiri [14] independently showed that β-continuity and almost quasicontinuity are equivalent of each other. Popa and Noiri [15] intoduced and investigated weakly *β*-continuous functions which are called weakly semi-precontinuous functions by Ghosh and Bhattacharyya [7]. Noiri and Popa [12] introduced and investigated almost *β*-continuity. The purpose of the present paper is to introduce and investigate a stronger form of *β*-continuity called strongly *θ-β*-continuous functions.

2. PRELIMINARIES

Throughout the present paper, X and Y denote topological spaces. Let S be a subset of X. We denote the interior and the closure of a set S by Int (S) and Cl(S), respectively. A subset S is said to be *β*-open [1] or *semi-preopen* [3] (resp. *α-open*[9]) if $S \subset \text{Cl}(\text{Int}(\text{Cl}(S)))$ (resp. $S \subset \text{Int}(\text{Cl}(\text{Int}(S)))$). The complement of a *β*-open (resp. semi-preopen) set is called *β*-closed (resp semi-preclosed). The intersection of all semi-preclosed sets containing S is called the semi-preclosure [3] of S and is denoted by spCl(S). The *semi-preinterior* of S is defined by the union of all *semi-preopen* sets contained in S and is denoted by spInt(S). The family of all semi-preopen sets of X is denoted by SPO(X). We set $\text{SPO}(X,x) = \{U : x \in U$ and $U \in \text{SPO}(X)\}$. A point x of X is called a *θ-cluster point* of S if $\text{Cl}(U) \cap S \neq \phi$ for every open set U of X containing x. The set of all *θ*-cluster points of S is called the *θ-closure* of S and is denoted by $\text{Cl}_\theta(S)$. A subset S is said to be *semi-pre-θ-closed* [16] if $S = \text{Cl}_\theta(S)$.

The complement of a θ-closed set is said to be *θ-open*. A point x of X is called a *semi-pre θ-cluster point* of S if $spCl(U) \cap S \neq$ for every semi-preopen set U of X containing x. The set of all semi-pre θ-cluster points of S is called the *semi-pre θ-closure* (briefly *sp-θ-closure* of S) and is denoted by $spCl_\theta(S)$. A subset S is said to be *semi-pre-θ-closed* (briefly *sp-θ-closed*) if $S = spCl_\theta(S)$. The complement of a *semi-pre-θ-closed* set is said to be *semi-pre-θ-open* (briefly *sp-θ-open*).

Definition 2.1. A function $f : X \to Y$ is said to be

(1) *β-continuous* [1] or *almost quasicontinuous* [5] if $f^{-1}(V) \in SPO(X)$ for each open set V or Y.

(2) *Weakly β-continuous* [15] (resp. *almost β-continuous* [12] if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in SPO(X,x)$ such that $f(U) \subset Cl(V)$ (resp. $f(U) \subset Int\ (Cl(V))$).

Definition 2.2. A function $f : X \to Y$ is said to be *strongly θ-β-continuous* (briefly *st.θ.β.c.*) if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in SPO(X,x)$ such that $f(spCl(U)) \subset V$.

Definition 2.3. A function $f : X \to Y$ is said to be *strongly-θ-continuous* [10] if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists an open neighbourhood U of x such that $f(Cl(U)) \subset V$.

Remarks 2.5. (1) Strong θ-β continuity is stronger than β-continuity and is weaker than strong θ-continuity.

(2) Strong θ-β-continuity and continuity are independent of each other as the following simple examples show.

Example 2.6. Let $X = \{a,b,c\}$, $\tau = \{\Phi, X, \{a,b\}\}$ and $\sigma = \{\Phi, X, \{c\}\}$. Define a function $f : (X, \tau) \to (X, \sigma)$ as follows : $f(a) = a$, $f(b) = f(c) = c$. Then f is *st.θ.β.c.* but it is not continuous.

Example 2.7. Let $X = \{a,b,c\}$ and $\tau = \{ \Phi, X\{a\}, \{a,b\}\}$, Then, the identity function $f : (X, \tau) \to (X,\tau)$ is continuous but not *st.θ.β.c.* at a.

3. CHARACTERIZATIONS

Theorem 3.1. *For a function $f : X \to Y$, The following properties are equivalent :*

(1) *f is strongly θ-β-continuous ;*

(2) *$f^{-1}(V)$ is sp-θ-open in X for every open set V of Y ;*

(3) *$f^{-1}(F)$ is sp-θ-closed in X for every closed set F of Y ;*

(4) $f(\mathrm{spCl}_\theta(A)) \subset \mathrm{Cl}(f(A))$ *for every subset* A *of* X ;

(5) $\mathrm{spCl}_\theta(f^{-1}(B)) \subset f^{-1}\mathrm{Cl}(f(B))$ *for every subset* B *of* Y.

Proof. (1) ⇒ (2) : Let V be any open set of Y. Suppose that $x \in f^{-1}(V)$. There exists $U \in$ SPO (X,x) such that $f(\mathrm{spCl}(U)) \subset V$. Therefore, we have $x \in U \subset \mathrm{spCl}(U) \subset f^{-1}(V)$. This shows that $f^{-1}(V)$ is sp-θ-open in X.

(2) ⇒ (3) : This is obvious.

(3) ⇒ (4) : Let A be any subset of X. Since $\mathrm{Cl}(f(A))$ is closed in Y, by (3) $f^{-1}(\mathrm{Cl}(f(A)))$ is sp-θ-closed and we have

$$\mathrm{spCl}_\theta(A) \subset \mathrm{spCl}_\theta(f^{-1}(f(A))) \subset \mathrm{spCl}_\theta(f^{-1}(\mathrm{Cl}(f(A))) = f^{-1}(\mathrm{Cl}(f(A))).$$

Therefore, we obtain $f(\mathrm{spCl}_\theta(A)) \subset \mathrm{Cl}((f(A))$.

(4) ⇒ (5) : Let B be any subset of Y. By (4), we obtain $f(\mathrm{spCl}_\theta(f^{-1}(B)))$ $\subset Cl(f(f^{-1}(B))) \subset \mathrm{Cl}(B)$ and hence $\mathrm{spCl}_\theta(f^{-1}(f(B)) \subset f^{-1}(\mathrm{Cl}(B))$.

(5) ⇒ (1) : Let $x \in X$ and V be any open neighborhood of $f(x)$. Since $Y - V$ is closed in Y, we have $\mathrm{spCl}_\theta(f^{-1}(Y - V)) \subset f^{-1}(\mathrm{Cl}(Y - V)) = f^{-1}(Y - V)$. Therefore, $f^{-1}(Y - V)$ is an *sp*-θ-closed in X and $f^{-1}(V)$ is an *sp*-θ-open set containing x. There exists $U \in$ SPO (X,x) such that $\mathrm{SpCl}(U) \subset f^{-1}(V)$; hence $f(\mathrm{spCl}(U)) \subset$ V. This shows that f is *st.θ.β*.c.

Definition 3.2. A function $f : X \to Y$ is said to be *faintly β-continuous* [11] if for each point $x \in X$ and each θ-open set V containin $f(x)$, there exists $U \in$ SPO (X,x) such that $f(U) \subset V$.

Theorem 3.3. *Let Y be a regular space. Then, for a function* $f : X \to Y$ *the following properties are equivalent* :

(1) f *is faintly β-continuous ;*

(2) f *is weakly β-continuous ;*

(3) f *is almost β-continuous ;*

(4) f *is β-continuous ;*

(5) f *is st.θ.β*.c.

Proof. It is shown in [11] that (1), (2) and (4) are equivalent. Since it is obvious that (5) implies (4), we shall show that (4) implies (5).

(4) $\Rightarrow$ (5) : Let $x \in X$ and V be an open set Y containing $f(x)$. Since Y is regular, there exists an open set W such that $f(x) \in W \subset \mathrm{Cl}(W) \subset V$. Since f is β-continuous, there exists $U \in \mathrm{SPO}(X,x)$ such that $f(U) \subset W$. We shall show that $f(\mathrm{spCl}(U)) \subset \mathrm{Cl}(W)$. Suppose that $y \notin \mathrm{Cl}(W)$. There exists an open neighborhood G of y such that $G \cap W = \Phi$. Since f is β-continuous, $f^{-1}(G) \in \mathrm{SPO}(X)$ and $f^{-1}(G) \cap U = \Phi$ and hence $f^{-1}(G) \cap \mathrm{SpCl}(U) = \Phi$. Therefore, we obtain $G \cap f(\mathrm{spcl}(U)) = \Phi$ and $y \notin f(\mathrm{spCl}(U))$. Consequently, we have $f(\mathrm{spCl}(U) \subset \mathrm{Cl}(W) \subset V$. This shows that f is st.$\theta.\beta$.c.

Definition 3.4. A space X is said to be *semipre-regular* (resp. *β-regular* [2] or *sp-regular* [13] if for each semi-preclosed (resp. closed) set F and each point $x \in X - F$, there exist disjoint semi-preopen sets U and V such that $x \in U$ and $F \subset V$.

Theorem 3.5. *A continuous function $f : X \rightarrow Y$ is st.$\theta.\beta$.c. if and only if X is β-regular.*

Proof. *Necessity.* Let $f : X \rightarrow Y$ be the identity function. Then f is continuous and *st.$\theta.\beta$.c.* by our hypothesis. For any open set U of X and any point x of U, we have $f(x) = x \in U$ and there exists $G \in \mathrm{SPO}\ (X,x)$ such that $f(\mathrm{spCl}(G) \subset \mathrm{U}$. Therefore, we have $x \in G \subset \mathrm{SpCl}(G) \subset \mathrm{U}$. It follows from Theorem 2.1 of [2] that X is β-regular.

Sufficiency, Suppose that $f : X \rightarrow Y$ is continuous and X is β-regular. For any $x \in X$ and any open neighborhood V of $f(x)$, $f^{-1}(V)$ is an open set of X containing x. Since X is β-regular, there exists $U \in \mathrm{SPO}\ (X)$ such that $x \in U \subset \mathrm{SpCl}(U) \subset f^{-1}(V)$ by Theorem 2.1 of [2]. Therefore, we have $f(\mathrm{spCl}(U)) \subset V$. This shows that f is *st.$\theta.\beta$.c.*

Theorem 3.6. *Let X be a semipre-regular space. Then $f : X \rightarrow Y$ is st.$\theta.\beta$.c. if and only if f is β-continuous.*

Proof. Suppose that f is β-continuous. Let $x \in X$ and V be any open set of Y containing $f(x)$. By the β-continuity of f, we have $f^{-1}(V) \in \mathrm{SPO}\ (X,x)$ and hence there exists $U \in \mathrm{SPO}\ (X,x)$ such that $\mathrm{spCl}(U) \in f^{-1}(V)$. Therefore, we obtain $f(\mathrm{spCl}(U)) \subset V$. This shows that f is *st.$\theta.\beta$.c.* The converse is obvious.

4. SOME PROPERTIES

Theorem 4.1. *Let $f : X \rightarrow Y$ be a function and $g : X \rightarrow X \times Y$ the graph function of f. Then, the following properties hold :*

(1) *If g is st.$\theta.\beta$.c., then f is st.$\theta.\beta$.c. and X is β-regular.*

(2) *If f is st.$\theta.\beta$.c. and X is semipre-regular, then g is st.$\theta.\beta$.c.*

Proof. (1) Suppose that g is *st.$\theta.\beta$.c.* First, we show that f is *st.$\theta.\beta$.c.* Let $x \in X$ and V be an open neighborhood of $f(x)$. Then $X \times V$ is an open set ot $X \times Y$ containing $g(x)$. Since g is *st.$\theta.\beta$.c.*, there exists $U \in SPO\ (X,x)$ such that $g(\mathrm{spCl}(U)) \subset X \times V$. Therefore, we obtain $f(\text{spCl}(U)) \subset V$. Next, we show that X is β-regular. Let U be any open set of X and $x \in U$. Since $g(x) \in U \times Y$ and $U \times Y$ is open is $X \times Y$, there exists $G \in$ SPO (X,x) such that $g(\text{spCl}(G)$ $U \times Y$. Therefore, we obtain $x \in G$ $\text{spCl}(G) \subset U$ and hence X is β-regular by Theorem 2.1 of [2].

(2) Let $x \in X$ and W be any open set of $X \times Y$ containing $g(x)$. There exist open sets $U_1 \subset X$ and $V \subset Y$ such that $g(x) = (x,f(x)) \in U_1 \times V \subset W$. Since f is *st.θ.β.c.*, there exists $U_2 \in$ SPO (X,x) such that $f(\text{spCl}(U_2) \subset V$. Since X is semipre-regular and $U_1 \cap U_2 \in \text{SPO}(X,x)$, there exists $U \in$ SPO (X,x) such that $x \in U \subset \text{spCl}(U) \subset U_1 \cap U_2$. Therefore, we obtain $g(\text{spCl}(U)) \subset U_1 \times f(\text{spCl}(U_2)) \subset U_1 \times V \subset W$. This shows that g is *st.θ.β.c.*

Corollary 4.2. *Let X be a semipre-regular space. Then, a function $f : X \to Y$ is st.θ.β.c. if and only if the graph function $g : X \to X \times Y$ is st.θ.β.c.*

Lemma 4.3. (Abd El-Monsef et. al. [1] *Let A and X_0 be subsets of a space X.*

(1) *If $A \in$ SPO (X) and Y is α-open in X, then $A \cap Y \in$ SPO (Y).*

(2) *If $A \in$ SPO (Y) and $Y \in$ SPO (X), then $A \in$ SPO (X).*

Lemma 4.4. *Let X be a topological space and A, Y subsets of X such that $A \subset Y \subset X$ and Y is α-open in X. Then the following properties hold :*

(1) *$A \in \text{SPO}(X)$ if and only $A \in$ SPO (X),*

(2) $\text{spCl}(A) \cap Y = \text{spCl}_Y(A)$, *where $\text{spCl}_Y(A)$ denotes the semipre-closure of A in the subspace Y.*

Proof. (1) Let $A \in \text{SPO}(Y)$. Since every α-open set is β-open, by Lemma 4.3, we have $A \in \text{SPO}(X)$. Conversely, let $A \in \text{SPO}(X)$. By Lemma 4.3, $A = A \cap Y \in \text{SPO}(Y)$.

(2) Let $x \in \text{spCl}(A) \cap Y$ and $V \in \text{SPO}(Y,x)$. Then, by (1) $V \in$ SPO (X,x) and hence $V \cap A \neq \phi$. Therefore, $x \in \text{spCl}_Y(A)$. Conversely, let $x \in \text{spCl}_Y(A)$ and $V \in$ SPO (X,x). Then by Lemma 4.3 $x \in V \cap Y \in \text{SPO}(Y)$ and hence $\phi \neq A \cap (V \cap Y) \subset A \cap V$. Therefore, we obtain $x \in \text{spCl}(A) \cap Y$.

Theorem 4.5. *If $f : X \to Y$ is st.θ.β.c. and X_0 is an α-open subset of X, then the restriction $f/X_0 : X_0 \to Y$ is st.θ.β.c.*

Proof. For any $x \in X_0$ and any open neighbourhood V of $f(x)$, there exists $U \in \text{SPO}(X,x)$ such that $f(\text{spCl}(U)) \subset V$ since f is *st.θ.β.c.* Put $U_0 = U \cap X_0$, then by Lemmas 4.3 and 4.4, $U_0 \in \text{SPO}(X_0,x)$ and $\text{sp}\,Cl_{X_0}(U_0) \subset \text{spCl}(U_0)$. Therefore, we obtain

$$(f/X_0)(\text{sp}\,Cl_{X_0}(U_0))) = f(\text{sp}\,Cl_{X_0}(U_0)) \subset f(\text{spCl}(U_0)) \subset f(\text{pCl}(U)) \subset V.$$

This shows that f/X_0 is *st.θ.β.c.*

Definition 4.6. A function $f : X \to Y$ is said to be

(1) *β-irresolute [8] if $f^{-1}(V) \in SPO(X)$ for each $V \in SPO(Y)$,*

(2) *pre-β-open [8] if $f(U) \in SPO(Y)$ for each $U \in SPO(X)$.*

Lemma 4.7. *If $f : X \to Y$ is β-irresolute and V is an sp-θ-open in Y, then $f^{-1}(V)$ is sp-θ-open in X.*

Proof. Let V be an sp-θ-open set of Y and $x \in f^{-1}(V)$. There exists $W \in SPO(Y)$ such that $f(x) \in W \subset spCl(W) \subset V$. Since f is β-irresolute, we have $f^{-1}(W) \in SPO(X)$ and $f^{-1}(spCl(W)) \in SPC(X)$. Therefore, we obtain $x \in f^{-1}(W) \subset spCl(f^{-1}(W)) \subset f^{-1}(spCl(W)) \subset f^{-1}(V)$. This shows that $f^{-1}(V)$ is sp-θ-open in X.

Theorem 4.8. *Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then, the following properties hold :*

(1) *If f is st.θ.β.c. and g is continuous, then the composition $g \circ f : X \to Y$ is st.θ.β.c.*

(2) *If f is β-irresolute and g is st.θ.β.c., then $g \circ f$ is st.θ.β.c.*

(3) *If $f : X \to Y$ is a pre-β-open bijection and $g \circ f : X \to Z$ is st.θ.β.c., then g is st.θ.β.c.*

Proof. (1) This is obvious from Theorem 3.1.

(2) This follows immediately from Theorem 3.1 and Lemma 4.7.

(3) Let W be any open set of Z. Since $g \circ f$ is *st.θ.β.c.*, $(g \circ f)^{-1}(W)$ *is sp-θ*-open in X. Since f is pre-β-open and bijective, f^{-1} is β-irresolute and by Lemma 4.7 we have $g^{-1}(W) = f((g \circ f)^{-1}(W))$ is sp-θ-open in Y. Hence, by Theorem 3.1 g is *st.θ.β.c.*

Let $\{X_\alpha : \alpha \in A\}$ be a family of topological spaces, A_α a nonempty subset of X_α for each $\alpha \in A$ And X = $\Pi\{X_\alpha : \alpha \in A\}$ denote the product space, where A is nonempty.

Lemma 4.9. (Abd El-Monsef [2] *Let n be a positive integer and*

$A = \prod_{j=1}^{n} A_{\alpha j} \times \prod_{\alpha \neq \alpha j} X_\alpha$ *Then the following properties hold :*

(1) $A \in$ SPO(X) if and only if $A_{\alpha_j} \in SPO(X_{\alpha_j})$ *for each j = 1, 2, ..., n.*

(2) $spCl\left(\prod_{\alpha \in A} A_\alpha\right) \subset \prod_{\alpha \in A} spCl(A_\alpha)$.

Theorem 4.10. *If a function* $f_\alpha : X_\alpha \to Y_\alpha$ *is st.θ.β.c. for each* $\alpha \in A$. *Then the product function* $f:\prod X_\alpha \to \prod Y_\alpha$, *defined by* $f(\{x_\alpha\}) = \{f_\alpha(x_\alpha)\}$ *for each* $x = \{x_\alpha\}$, *is st.θ.β.c.*

Proof. Let $x = \{x_\alpha\} \in \prod X_\alpha$ and W be any open set of $\prod Y_\alpha$ containing $f(x)$. Then there exists an open set V_{α_j} of Y_{α_j} such that

$$f(x)=\{f_\alpha(x_\alpha)\}\in\prod_{j=1}^{n} V_{\alpha j}\times\prod_{\alpha\neq\alpha j} Y_\alpha \subset W.$$

Since f_α is *st.θ.β.c.* for each α, there exists $U_{\alpha j}\in SPO(X_{\alpha j},X_{\alpha j})$ such that $f_{\alpha j}(spCl(U_{\alpha j}))\subset V_{\alpha j}$ for $j = 1, 2, \ldots, n$. Now, put $U = \prod_{j=1}^{n} U_{\alpha j}\times\prod_{\alpha\neq\alpha j} X_\alpha$. Then, it follows from Lemma 4.9 that U ∈ SPO (ΠX_α,x). Moreover, we have

$$f(\mathrm{spCl}(U)) \subset f\Big(\prod_{j=1}^{n} spCl(U_{\alpha j})\times\prod_{\alpha\neq\alpha j} X_\alpha\Big)\subset\prod_{j=1}^{n} f_{\alpha j}(spCl(U_{\alpha j}))\times\prod_{\alpha\neq\alpha j} Y_\alpha \subset$$

$$\prod_{j=1}^{j} V_{\alpha j}\times\prod_{\alpha\neq\alpha j} Y_\alpha \subset W$$

This shows that f is *st.θ.β.c.*

5. *St.θ.β.c.* FUNCTIONS AND SEPARATIONS AXIOMS

A space X is said to be *sp*-T_2 or β-T_2 [8] if for each pair of distinct points x and y in X, there exist $U \in$ SPO(X,x) and $V \in$ SPO(X,y) such that $U \cap V = \Phi$.

Theorem 5.1. *If* $f : X \to Y$ *is a st.θ.β.c. injection and* Y *is* T_0, *then* X *is sp*-T_2.

Proof. Suppose that Y is T_0. Let x and y be any distinct points of X. Since f is injective, $f(x) \neq f(y)$ and there exists either an open neighbourhood V of $f(x)$ not containing $f(y)$ or an open neighbourhood W of $f(y)$ not containing $f(x)$. If the first case holds, then there exists U ∈ SPO(X,x) such that f(spCl(U)) ⊂ V. Therefore, we obtain $f(y) \notin f$(spCl(U)) and hence X—spCl(U) ∈ SPO(X,y). If the second case holds, then we obtain the similar result. Therefore, X is *sp*-T_2.

Theorem 5.2. *If* $f : X \to Y$ *is a st.θ.β.c. function and* Y *is Hausdorff, then a subset* $E = \{(x,y) : f(x) = f(y)\}$ *is sp-θ-closed in* $X \times X$.

Proof. Suppose that $(x,y) \notin E$. It follows that $f(x) \neq f(y)$. Since Y is Hausdorff, there exist disjoint open sets V and W in Y containing $f(x)$ and $f(y)$, respectively. Since f is *st.θ.β.c.*, there exist $U \in$ SPO(X,x) and $G \in$ SPO(X,y) such that f(spCl(U)) ⊂ V and f(spCl(G)) ⊂ W. Set $D = U \times G$. It follows that $(x,y) \in D \in$ SPO($X \times X$) and spCl(D) ∩ E ⊂ [spCl(U) × spCl(G)] ∩ E = Φ. Therefore, E is *sp*-θ-closed in $X \times X$.

For a function $f : X \to Y$ the subset $\{(x,f(x)) : x \in X\}$ of $X \times Y$ is called the *graph* of f and is denoted by $G(f)$.

Definition 5.3. The graph $G(f)$ of a function $f : X \to Y$ is said to be *strongly sp-closed* if for each $(x,y) \in (X \times Y) - G(f)$, there exist $U \in SPO(X,x)$ and an open set V in Y containing y such that $(SpCl(U)) \times V) \cap G(f) = \Phi$.

Lemma 5.4. *The graph $G(f)$ of a function $f : X \to Y$ is strongly sp-closed in $X \times Y$ if and only if for each point $(x,y) \in (X \times Y) - G(f)$, there exist $U \in SPO(X,x)$ and an open set V in Y containing y such that $f(spCl(U)) \cap V = \Phi$.*

Theorem 5.5. *If $f : X \to Y$ is st.θ.β.c. and Y is Hausdorff, then $G(f)$ is strongly sp-closed in $X \times Y$.*

Proof. Let $(x,y) \in (X \times Y) - G(f)$. It follows that $f(x) \neq y$. Since Y is Hausdorff, there exist disjoint open sets V and W in Y containing $f(x)$ and y, respectively. Since f is *st.θ.β.c.*, there exists $U \in SPO(X,x)$ such that $f(spCl(U)) \subset V$. Therefore, $f(spCl(U)) \cap W = \Phi$. and $G(f)$ is strongly *sp*-closed in $X \times Y$.

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