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ON RELATIVE DEFECTS OF MEROMORPHIC FUNCTIONS

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ABSTRACT : In the paper we consider two different meromorphic functions having common roots and find some relations involving the relative defects. We also study the relationship between the usual defect and the relative defect of a meromorphic function corresponding to distinct zeros and distinct poles with the Nevanlinna defect.

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1. INTRODUCTION AND NOTATIONS

Let f_1 and f_2 be two non-constant meromorphic functions defined in the open complex plane C. Let $n_0(r,a)$ denoted the number of common roots in the disk $|z| \leq r$ of the two equations $f_1 = a$ and $f_2 = a$ where a is any complex number and $\bar{n}_0(r,a)$ denotes the number of distinct common roots in the disk $|z| \leq r$ of the two equations $f_1 = a$ and $f_2 = a$. A. P. Singh [3] found some relations on relative defects corresponding to the common roots of two meromorphic functions. In the paper we further investigate the results of Singh [3] and prove some new results on relative defects of the common roots of $f_1 = a$ and $f_2 = a$.

To start our paper we require the following :

$$\text{Let } \bar{N}_0(r,a) = \int_0^r \frac{\bar{n}_0(t,a) - \bar{n}_0(0,a)}{t} dt + \bar{n}_0(0,a) \log r$$

$$\bar{N}_{1,2}(r,a) = \bar{N}(r, \frac{1}{f_1 - a}) + \bar{N}(r, \frac{1}{f_2 - a}) - 2\bar{N}_0(r,a).$$

Also let $\bar{n}_0^{(k)}(r,a)$, $\bar{N}_{1,2}^{(k)}(r,a)$ etc. denote the corresponding quantities with respect to $f_1^{(k)}$ and $f_2^{(k)}$, where k is any non negative integer.

Now we set the following quantities :

$$\delta_{1,2}(a) \quad = \quad 1 - \limsup_{r \to \infty} \frac{N_{1,2}(r,a)}{T(r,f_1) + T(r,f_2)}$$

$$\delta_{1,2}^{(k)}(a) = 1-\limsup_{r\to\infty}\frac{N_{1,2}^{(k)}(r,a)}{T(r,f_1)+T(r,f_2)}$$

$$\delta_0(a) = 1-\limsup_{r\to\infty}\frac{N_0(r,a)}{T(r,f_1)+T(r,f_2)}$$

$$\Theta_{1,2}(a) = 1-\limsup_{r\to\infty}\frac{\overline{N}_{1,2}(r,a)}{T(r,f_1)+T(r,f_2)}$$

$$\Theta_{1,2}^{(k)}(a) = 1-\limsup_{r\to\infty}\frac{\overline{N}_{1,2}^{(k)}(r,a)}{T(r,f_1)+T(r,f_2)}$$

$$\Theta_0(a) = 1-\limsup_{r\to\infty}\frac{\overline{N}_0(r,a)}{T(r,f_1)+T(r,f_2)}$$

Let $\alpha \in C \cup \{\infty\}$ For a non-constant meromorphic function f, the Nevanlinna defect $\delta(\alpha, f)$ of α is defined in the following manner :

$$\delta(\alpha;f) = 1-\limsup_{r\to\infty}\frac{N(r,\alpha;f)}{T(r,f)} = \liminf_{r\to\infty}\frac{m(r,\alpha;f)}{T(r,f)}$$

The term $1-\limsup_{r\to\infty}\frac{\overline{N}(r,\alpha;f)}{T(r,f)}$ is called the usual defect of α corresponding to distinct zeros and is denoted by $\Theta(\alpha, f)$. The usual defect of α corresponding to distinct poles is similarly defined. A. P. Singh [2] introduced the term relative defect for distinct zeros and poles and established various relations between it, relative defects and the usual defects. In the paper we establish some results on the relationship between Nevanlinna defect, usual and relative defect corresponding to distinct zeros and poles in the line of Singh [2].

The following definition is well known.

Definition 1.1. The relative defect of 'α' with respect to $f^{(k)}$ corresponding to simple zeros is defined as

$$\Theta_r^{(k)}(\alpha;f) = 1-\limsup_{r\to\infty}\frac{\overline{N}(r,\alpha;f^{(k)})}{T(r,f)} \quad \text{for } k = 1, 2, 3, \ldots$$

We do not explain the definitions and standard notations of Nevanlinna theory because those are available in [1]. The term S(r,f) denotes any quantity satisfying $S(r,f) = o\{T(r,f)\}$

as $r\to\infty$ through all values of r if f is of finite order and except possibly for a set of r of finite linear measure otherwise.

2. LEMMA

In this section we present a lemma which will be needed in the sequel.

The following lemma is due to Millox {p.55, [1]}.

Lemma 2.1.[1]. Let k be a positive integer and $\psi=\sum_{i=0}^{k} a_i f^{(i)}$ where a_i are meromorphic functions such that $T(r,a_i) = S(r,f)$. Then $m\left(r,\frac{\psi}{f}\right)=S(r,f)$

3. THEOREMS

In this section we present the main results of the paper.

Theorem 3.1. Let f_1 and f_2 be any two meromorphic functions such that $\overline{N}(r,f_1) = S(r,f_1)$ and $\overline{N}(r,f_2) = S(r,f_2)$. Also let a_i (i = 1, 2, ...p) and b_j (j = 1, 2, ... q) be two sets of distinct finite non zero complex numbers. Then for any positive integer k.

$$\Theta_{1,2}(0)+q\delta_{1,2}(0)+\sum_{i=1}^{p}\Theta_{1,2}(a_i)+\sum_{j=1}^{q}\Theta_{1,2}{}^{(k)}(b_j)+$$

$$2\left\{\Theta_0(a)+q\delta_0(a)+\sum_{i=1}^{p}\Theta_0(a_i)+\sum_{i=1}^{q}\Theta_0^{(k)}(b_j)\right\}\le 3+2p+5q.$$

Proof. From Nevanlinna's first fundamental theorem and by Lemma 2.1 we get,

$$T(r,f)=T(r,\frac{1}{f})+O(1)\le N(r,\frac{1}{f})+m(r,\frac{f^{(k)}}{f})+m(r,\frac{1}{f^{(k)}})+O(1)$$

Thus $T(r,f)\le N(r,\frac{1}{f})+T(r,\frac{1}{f^{(k)}})-N(r,\frac{1}{f^{(k)}})+S(r,f)$

i.e., $T(r,f)\le N(r,\frac{1}{f})+T(r,f^{(k)})-N(r,\frac{1}{f^{(k)}})+S(r,f)$ ... (1)

Again by Nevanlinna's second fundamental theorem,

$$qT(r,f^{(k)}) \le \overline{N}(r,f^{(k)}) + \sum_{j=1}^{q} \overline{N}(r,\frac{1}{f^{(k)}-b_j}) + \overline{N}(r,\frac{1}{f^{(k)}}) + S(r,f) \quad \dots (2)$$

Therefore from (1) and (2) we obtain,

$$qT(r,f) \le \overline{N}(r,f^{(k)}) + \overline{N}(r,\frac{1}{f^{(k)}}) - qN(r,\frac{1}{f^{(k)}}) + \sum_{j=1}^{q} \overline{N}(r,\frac{1}{f^{(k)}-b_j})$$

$$+qN(r;\frac{1}{f}) + S(r,f) \quad \dots (3)$$

Since $\overline{N}(r,f^{(k)}) = \overline{N}(r,f)$ and $\overline{N}(r,\frac{1}{f^{(k)}}) - qN(r,\frac{1}{f^{(k)}}) \le 0$ it follows from (3),

$$qT(r,f) \le \sum_{j=1}^{q} \overline{N}(r,\frac{1}{f^{(k)}-b_j}) + \overline{N}(r,f) + qN(r,\frac{1}{f}) + S(r,f) \quad \dots (4)$$

In view of Nevanlinna's second fundamental theorem,

$$pT(r,f) \le \overline{N}(r,f) + \overline{N}(r,\frac{1}{f}) + \sum_{i=1}^{p} \overline{N}(r,\frac{1}{f-a_i}) + S(r,f) \quad \dots (5)$$

Adding (4) and (5) it follows that,

$$(p+q)T(r,f) \le \overline{N}(r,\frac{1}{f}) + qN(r,\frac{1}{f}) + 2\overline{N}(r,f) + \sum_{i=1}^{p} \overline{N}(r,\frac{1}{f-a_i})$$

$$+\sum_{j=1}^{q} \overline{N}(r,\frac{1}{f^{(k)}-b_j}) + S(r,f) \quad \dots (6)$$

Applying this inequality for f_1 and f_2 we get,

$(p + q)\{T(r,f_1) + T(r,f_1)\}$

$$\le \left\{\overline{N}(r,\frac{1}{f_1}) + \overline{N}(r,\frac{1}{f_2})\right\} + q\left\{N(r,\frac{1}{f_1}) + N(r,\frac{1}{f_2})\right\} + 2\{\overline{N}(r,f_1) + \overline{N}(r,f_2)\}$$

$$\sum_{i=1}^{p}\left\{\overline{N}(r,\frac{1}{f_1-a_i}) + \overline{N}(r,\frac{1}{f_2-a_i})\right\} + \sum_{j=1}^{q}\left\{\overline{N}(r,\frac{1}{f_1^{(k)}-b_j}) + \overline{N}(r,\frac{1}{f_2^{(k)}-b_j})\right\}$$

$+ S(r,f_1) + S(r,f_2)$

$$+\sum_{i=1}^{p}\{\overline{N}_{1,2}(r,0)+2\overline{N}_0(r,a_i)\}+\sum_{j=1}^{q}\{(N_{1,2}^{(k)}(r,b_j)+2\overline{N}_0(r,b_j)\}+S(r,f_1)+S(r,f_2)$$

$$+\sum_{i=1}^{p}\{\overline{N}_{1,2}(r,a_i)+2\overline{N}_0(r,a_i)\}+\sum_{j=1}^{q}\{N_{1,2}^{(k)}(r,b_j)+2\overline{N}_0(r,b_j)\}+S(r,f_1)+S(r,f_2) \quad \ldots (7)$$

Now dividing both sides of (7) by $T(r,f_1) + T(r,f_2)$ and taking limit superior we get,

$$(p+q)\le\{1-\Theta_{1,2}(0)\}+2\{1-\Theta_0(a)\}+q\{1-\delta_{1,2}(0)\}+2q\{1-\delta_0(a)\}$$

$$+\sum_{i=1}^{p}\{1-\Theta_{1,2}(a_i)\}+2\sum_{i=1}^{p}\{1-\Theta_0(a_i)\}+\sum_{j=1}^{q}\{1-\Theta_{1,2}^{(k)}(b_j)\}+2\sum_{j=1}^{q}\{1-\Theta_0^{(k)}(b_j)\}$$

i.e., $p+q\le 3-\Theta_{1,2}(0)-2\Theta_0(a)+3q-q\delta_{1,2}(0)-2q\delta_0(a)+3p-$

$$\sum_{i=1}^{p}\Theta_{1,2}(a_i)\}-2\sum_{i=1}^{p}\Theta_0(a_i)+3q-\sum_{j=1}^{q}\Theta_{1,2}^{(k)}(b_j)\}-2\sum_{j=1}^{q}\Theta_0^{(k)}(b_j)$$

i.e.,

$$\Theta_{1,2}(0)+q\delta_{1,2}(0)+\sum_{i=1}^{p}\Theta_{1,2}(a_i)+\sum_{j=1}^{q}\Theta_{1,2}^{(k)}(b_j)+$$

$$2\{\Theta_0(a)+q\delta_0(a)+\sum_{i=1}^{p}\Theta_0(a_i)+\sum_{j=1}^{q}\Theta_0^{(k)}(b_j)\}\le 3+2p+5q$$

This proves the theorem.

Theorem 3.2. For any two meromorphic function f_1 and f_2 with $\overline{N}(r,f_1) = S(r,f_1)$ and $\overline{N}(r,f_2) = S(r,f_2)$

$$\Theta_{1,2}^{(k)}(0)+\sum_{j=1}^{q}\Theta_{1,2}^{(k)}(b_j)+q\sum_{i=1}^{p}\delta_{1,2}(a_i)+2\{\Theta_0^{(k)}(0)+\sum_{j=1}^{q}\Theta_0^{(k)}(b_j)+q\sum_{i=1}^{p}\delta_0(a_i)\}$$

$$\le 3+3q+2pq$$

where a_i ($i = 1, 2, \ldots\ldots p$) and b_j ($j = 1, 2, \ldots q$) be any two sets of distinct finite non zero complex numbers and k is any positive integer.

Proof. Let us consider $F=\sum_{i=1}^{p}\frac{1}{f-a_i}$

Then by Lemma 1 we obtain,

$$\sum_{i=1}^{p} m\left(r, \frac{1}{f-a_i}\right) \le m(r,F) + O(1)$$

i.e.,
$$\sum_{i=1}^{p} m\left(r, \frac{1}{f-a_i}\right) \le m(r, Ff^{(k)}) + m\left(r, \frac{1}{f^{(k)}}\right) + O(1)$$

i.e.,
$$\sum_{i=1}^{p} m\left(r, \frac{1}{f-a_i}\right) \le \sum_{i=1}^{p} m(r, \frac{f^{(k)}}{f-a_i}) + m\left(r, \frac{1}{f^{(k)}}\right) + O(1)$$

i.e.,
$$\sum_{i=1}^{p} m\left(r, \frac{1}{f-a_i}\right) \le m\left(r, \frac{1}{f^{(k)}}\right) + S(r,f) \qquad \ldots (8)$$

Adding to $\sum_{i=1}^{p} N\left(r, \frac{1}{f-a_i}\right)$ to both sides of (8) we get,

$$\sum_{i=1}^{p} T\left(r, \frac{1}{f-a_i}\right) \le T\left(r, \frac{1}{f^{(k)}}\right) + \sum_{i=1}^{p} N\left(r, \frac{1}{f-a_i}\right) + S(r,f)$$

i.e.,
$$pqT(r,f) \le qT(r,f^{(k)}) + q\sum_{i=1}^{p} N(r, \frac{1}{f-a_i}) + S(r,f) \qquad \ldots (9)$$

Again by Nevanlinna's second fundamental theorem and in view of $\overline{N}(r, f^{(k)}) = \overline{N}(r,f)$ we obtain,

$$qT(r,f^{(k)}) \le \overline{N}(r,f) + \overline{N}(r, \frac{1}{f^{(k)}}) + \sum_{j=1}^{q} \overline{N}\left(r, \frac{1}{f-b_j}\right) + S(r,f) \qquad \ldots (10)$$

Now from (9) and (10) it follows that,

$$pq\, T(r,f) \le \overline{N}(r,f) + \overline{N}\left(r, \frac{1}{f^{(k)}}\right) + q\sum_{i=1}^{p} N\left(r, \frac{1}{f-a_j}\right) +$$

$$\sum_{j=1}^{q} \overline{N}\left(r, \frac{1}{f^{(k)}-b_j}\right) + S(r,f) \qquad \ldots (11)$$

Applying (11) for f_1 and f_2 we get in view of $\overline{N}(r,f_1) = S(r,f_1)$ and $\overline{N}(r,f_2) = S(r,f_2)$ $pq\{T(r,f_1) + T(r,f_2)\}$

$$\leq \left\{\overline{N}\left(r,\frac{1}{f_1^{(k)}}\right)+\overline{N}\left(r,\frac{1}{f_2^{(k)}}\right)\right\}+\sum_{j=1}^{q}\left\{\overline{N}\left(r,\frac{1}{f_1^{(k)}-b_j}\right)+\overline{N}\left(r,\frac{1}{f_2^{(k)}-b_j}\right)\right\}$$

$$+\ q\sum_{i=1}^{p}\left\{N\left(r,\frac{1}{f_1-a_i}\right)+\overline{N}\left(r,\frac{1}{f_2-a_i}\right)\right\} + S(r,f_1) + S(r,f_2).$$

$$= \{\overline{N}_{1,2}^{(k)}(r,0)+2\overline{N}_0^{(k)}(r,0)\}+\{\overline{N}_{1,2}^{(k)}(r,b_j)+2\overline{N}_0^{(k)}(r,b_j)\}$$

$$+q\sum_{i=1}^{p}\{N_{1,2}(r,a_i)+2N_0(r,a_i)\}+S(r,f_1)+S(r,f_2) \quad \ldots (12)$$

Dividing both sides of (12) by $T(r,f_1) + T(r,f_2)$ and taking limit superior we obtain,

$$pq\leq\{1-\Theta_{1,2}^{(k)}(0)\}+2\{1-\Theta_0^{(k)}(0)\}+\sum_{j=1}^{q}\{1-\Theta_{1,2}^{(k)}(b_j)\}$$

$$+\ 2\sum_{j=1}^{q}\{1-\Theta_0^{(k)}(b_j)\}+q\sum_{i=1}^{p}\{1-\delta_{1,2}(a_i)\}+2q\sum_{i=1}^{p}\{1-\delta_0(a_i)\}.$$

i.e., $$pq\leq 3-\Theta_{1,2}^{(k)}(0)+2\Theta_0^{(k)}(0)+3q-\sum_{j=1}^{q}\Theta_{1,2}^{(k)}(b_j)-\sum_{j=1}^{q}\Theta_0^{(k)}(b_j)$$

$$+3pq-q\sum_{i=1}^{p}\delta_{1,2}(a_i)-2q\sum_{i=1}^{p}\delta_0(a_i)$$

i.e., $$\Theta_{1,2}^{(k)}(0)+\sum_{j=1}^{q}\Theta_{1,2}^{(k)}(b_j)+q\sum_{i=1}^{p}\delta_{1,2}(a_i)+$$

$$2\left\{\Theta_0^{(k)}(0)+\sum_{j=1}^{q}\Theta_0^{(k)}(b_j)+q\sum_{i=1}^{p}\delta_0(a_i)\right\}\leq 3 + 3q + 2pq.$$

Thus the theorem is proved.

Theorem 3.3. Let a_i (i = 1, 2, ...p) and b_j = (1, 2, ... q) be any two sets of finite non zero distinct complex numbers. Then for any two meromorphic functions f_1 and f_2 with

$$\overline{N}(r,f_1) = S(r,f_1),\ \overline{N}(r,f_2) = S(r,f_2)$$

$$\Theta_{1,2}^{(k)}(0)+2\Theta_0^{(k)}(0)+\Theta_{1,2}(0)+2\Theta_0(r,0)+q\delta_{1,2}(0)+2q\delta_0(0)+(1+q)\sum_{i=1}^{p}\delta(a_i)$$

$$+\ 2(1+q)\sum_{i=1}^{p}\Theta_0(a_i)+2\sum_{j=1}^{q}\Theta_{1,2}^{(k)}(b_j)+4\sum_{j=1}^{q}\Theta_0^{(k)}(b_j)\le 6 + 2p + 8q + 2pq$$

where k is any positive integer.

Proof. Adding (6) and (11) we get,

(p + q + pq) T(r,f)

$$\le 3\overline{N}(r,f)+\overline{N}\left(r,\frac{1}{f^{(k)}}\right)+\overline{N}\left(r,\frac{1}{f}\right)+qN\left(r,\frac{1}{f}\right)$$

$$+(1+q)\sum_{i=1}^{p}N\left(r,\frac{1}{f-a_i}\right)+2\sum_{j=1}^{q}\overline{N}\left(r,\frac{1}{f^{(k)}-b_j}\right)+S(r,f) \quad \dots (13)$$

Applying this inequality for f_1 and f_2 it follows that,

$(p + q + pq)\{T(r,f_1) + T(r,f_2)\}$

$$\le\left\{\overline{N}\left(r,\frac{1}{f_1^{(k)}}\right)+\overline{N}\left(r,\frac{1}{f_2^{(k)}}\right)\right\}+\left\{\overline{N}\left(r,\frac{1}{f_1}\right)+\overline{N}\left(r,\frac{1}{f_2}\right)\right\}$$

$$+q\left\{N\left(r,\frac{1}{f_1}\right)+N\left(r,\frac{1}{f_2}\right)\right\}+(1+q)\sum_{i=1}^{p}\left\{N\left(r,\frac{1}{f_1-a_i}\right)+N\left(r,\frac{1}{f_2-a_i}\right)\right\}$$

$$+2\sum_{i=1}^{q}\left\{N\left(r,\frac{1}{f_1^{(k)}-b_j}\right)+\overline{N}\left(r,\frac{1}{f_1^{(k)}-b_j}\right)\right\} + S(r,f_1) + S(r,f_2)$$

$$=\{\overline{N}_{1,2}^{(k)}(r,0)+2\overline{N}_0^{(k)}(r,0)\}+\{\overline{N}_{1,2}(r,0)+2\overline{N}_0(r,0)\}+$$

$$q\{N_{1,2}(r,0)+2N_0(r,0)\}+(1+q)\sum_{i=1}^{p}\{N_{1,2}(r,a_i)+2\overline{N}_0(r,a_i)\}$$

$$+2\sum_{j=1}^{q}\{\overline{N}_{1,2}^{(k)}(r,b_j)+2\overline{N}_0^{(k)}(r,b_j)\} + S(r,f_1) + S(r,f_2) \qquad \ldots (14)$$

On dividing both sides of (14) by $T(r,f_1) + T(r,f_2)$ and taking limit superior we get,

$$p + q + pq \le \{1-\Theta_{1,2}^{(k)}(0)\}+2\{1-\Theta_0^{(k)}(0)\}+\{1-\Theta_{1,2}^{(k)}(0)\}+2\{1-\Theta_0^{(k)}(r,0)\}$$

$$+q\{1-\delta_{1,2}(0)\}+2q\{1-\delta_0(0)\}+(1+q)\left\{p-\sum_{i=1}^{p}\delta(a_i)\right\}$$

$$+2(1+q)\left\{p-\sum_{i=1}^{p}\Theta_0(a_i)\right\}+2\left\{q-\sum_{j=1}^{q}\Theta_{1,2}^{(k)}(b_j)\right\}+\left\{4q-\sum_{j=1}^{q}\Theta_0^{(k)}(b_j)\right\}$$

i.e.,

$$\Theta_{1,2}^{(k)}(0)+2\Theta_0^{(k)}(0)+\Theta_{1,2}(0)+2\Theta_0(r,0)+q\delta_{1,2}(0)+2q\delta_0(0)$$

$$+(1+q)\sum_{i=1}^{p}\delta(a_i)+2(1+q)\sum_{i=1}^{p}\Theta_0(a_i)+2\sum_{j=1}^{q}\Theta_{1,2}^{(k)}(b_j)+4\sum_{j=1}^{q}\Theta_0^{(k)}(b_j)$$

$$\le 6+2p+8q+2pq.$$

This proves the theorem.

The next theorem is based on the relationship between the usual defect and the relative defect of a meromorphic function corresponding to distinct zeros and distinct poles with the Nevanlinna defect.

Theorem 3.4. Let f ba a meromorphic function and a_i (i = 1, 2, ...p), b_j (j = 1, 2, ... q) are finite distinct complex numbers such that $a_i \neq 0$, $b_j \neq 0$. Then for any positive integer k,

$$3\,\Theta(\infty,f)+\Theta_r^{(k)}(0,f)+\Theta(0,f)\; q\;\delta(0,f)+$$

$$(1+q)\sum_{i=1}^{p}\delta(a_i,f)+2\sum_{j=1}^{q}\Theta_r^{(k)}(b_j,f)\le 2q+5.$$

Proof. From (13) we obtain

$$p + q + pq \le 3\ \limsup_{r\to\infty}\frac{\overline{N}(r,f)}{T(r,f)}+\limsup_{r\to\infty}\frac{\overline{N}\left(r,\frac{1}{f^{(k)}}\right)}{T(r,f)}$$

$$+\limsup_{r\to\infty}\frac{\overline{N}\left(r,\frac{1}{f}\right)}{T(r,f)}+q\limsup_{r\to\infty}\frac{N\left(r,\frac{1}{f}\right)}{T(r,f)}$$

$$+(1+q)\sum_{i=1}^{p}\limsup_{r\to\infty}\frac{N\left(r,\frac{1}{f-a_i}\right)}{T(r,f)}$$

$$+2\sum_{j=1}^{q}\limsup_{r\to\infty}\frac{\overline{N}\left(r,\frac{1}{f^{(k)}-b_j}\right)}{T(r,f)}$$

i.e., p + q + pq

$$\le 3\{1-\Theta(\infty,f)\}+\{1-\Theta_r^{(k)}(0,f)\}+\{1-\Theta(0,f)\}+q\{1-\delta(0,f)\}+$$

$$(1+q)\{p-\sum_{i=1}^{p}\delta(a_i,f)\}+2\{q-\sum_{j=1}^{q}\Theta_r^{(k)}(b_j,f)\}$$

i.e., $3\Theta(\infty,f)+\Theta_r^{(k)}(0,f)+\Theta(0,f)+q\delta(0,f)+$

$$(1+q)\sum_{i=1}^{p}\delta(a_i,f)+2\sum_{j=1}^{q}\Theta_r^{(k)}(b_j,f)\le 2q+5 \qquad \ldots (15)$$

This proves the theorem.

The following corollary is a consequence of Theorem 3.4.

Corollary 3.5. If f is an entire function such that $\sum_{i=1}^{p}\delta(a_i,f)=1$, then for any positive integer k,

$$\Theta_r^{(k)}(0,f)+\Theta(0,f)+q\ \delta(0,f)+2\sum_{j=1}^{q}\Theta_r^{(k)}(b_j,f)\le q+1.$$

where a_i and b_j have the same meaning as in Theorem 3.4.

Proof. Since f is entire function $\Theta(\infty,f)=1$.

Now in view of $\sum_{i=1}^{p}\delta(a_i,f)=1$ and $\Theta(\infty,f)=1$ it follows from (15),

$$3+\Theta_r^{(k)}(0,f)+\Theta(0,f)+q\,\delta(0,f)+(1+q)$$

$$+2\sum_{i=1}^{q}\Theta_r^{(k)}(b_j,f)\leq 2q+5$$

from which the corollary follows.

REFERENCES

1. W. K. Hayman, Meromorphic Functions, The Clarendon Press, London (1964).
2. A. P. Singh, Relative defects of meromorphic functions, J. Indian Math. Soc. **44** (1980), pp 191-202.
3. A. P. Singh, Relative defects corresponding to the common roots of two meromorphic functions, Kodai Math. J., **6** (1983), pp 333-339.

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