Jour. Pure Math., Vol. 14, 1997, pp. 18-25

ON SEVERAL CLASSES OF ORTHODOX Γ — SEMIGROUPS

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ABSTRACT : In this paper the concepts of orthodox Γ—regular semigroups, orthogroup Γ—semigroups, and rectangular Γ—groups are introduced and studied. Some characterizations of these Γ—semigroups are obtained. Also the structure theorm of orthogroup Γ—semigroups is given. These notions are generalization of the corresponding ones in semigroups.

Key word : Union of groups, union of Γ—groups, orthodox semigroups, orthodox Γ—somigroups

1991 AMS subject classification : 20M 19

* This paper is written in Kunming while visiting Yunnan University.

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1. PRELIMINARIES

A set M is said to be a Γ—semigroup (due to Sen and Saha [1]) if the following conditions are satisfied :

(i) M = {a,b,c...} and Γ= {α,β,γ...} are two nonempty sets.

(ii) $a\alpha b \in M$ for all a, b ∈ M and all $\alpha \in \Gamma$.

(iii) $(a\alpha b)\beta c = a\alpha(b\beta c)$ for all a, b, c ∈ M and all α, β ∈ Γ.

Let M be a Γ— semigroup and α an element of Γ. We define the product a.b = $a\alpha b$, then (M, .) is a semigroup, denoted by M_α, called a related semigroup of M. If some one (so every, due to [1]) related semigroup of M is a group, then we say M is a Γ-group. An element a of M is said to be regular if a ∈ aΓMΓa, where aΓaMΓa = {$(a\alpha b)\beta a$: = b ∈ M, α,β ∈ Γ}. A Γ— semigroup is said to be regular if every element of M is regular (due to [1]. Binary relations $\mathcal{J}$, $\mathcal{L}$, $\mathcal{R}$, $\mathcal{H}$ and $\mathcal{D}$ on M (due to [7]). which are analogous of Green's relations in a semigroup, are defined as follows :

(i) $a \mathscr{I} b$ iff $(a) = (b)$,
(ii) $a \mathscr{L} b$ iff $(a)_1 = (b)_1$,
(iii) $a \mathscr{R} b$ iff $(a)_r = (b)_r$,
(iv) $a \mathscr{H} b$ iff $a \mathscr{L} b$ and $a \mathscr{R} b$
(v) $a \mathscr{D} b$ iff there exists $c \in M$ such that $a \mathscr{L} c$ and $c \mathscr{R} b$.

Where $(a)_r = a \Gamma M \cup \{a\}$, $(a)_l = M \Gamma a \cup \{a\}$, $(a) = a \Gamma M \cup M \Gamma a \cup M\Gamma a\Gamma M \cup \{a\}$. Throughout this paper E_α denotes the set $\{e : e\alpha e = e, e \in M\}$ and $V_\alpha^\beta(a)$ denotes the set $\{b : b \in M, a\alpha b\beta a = a \text{ and } b\beta a\alpha b = b\}$ for all $a \in M$ and all $\alpha, \beta \in \Gamma$. An element of E_α is said to be an α—idempotent of M.

The following known results will be used in this paper. A Γ— semigroup is said to be a Γ— regular semigroup if its every related semigroup is regular.

Lemma. 11. (due to [3]) A regular Γ— semigroup is a Γ— regular semigroup if and only if $V_\alpha^\beta(e) \neq \phi$ and $V_\beta^\alpha(e) \neq \phi$ for all $\alpha, \beta \in \Gamma$ and for all $e \in E_\alpha$.

Lemma. 12. (due to [3]) In a Γ — semigroup M for all $a \in M$ and all $\alpha, \beta \in \Gamma$, $V_\alpha^\beta(a) \neq \phi$ if and if M is regular, and $V_\alpha^\beta(e) \neq \phi$ and $V_\beta^\alpha(e) \neq \phi$ for all $e \in E_\alpha$.

Let M be a Γ—semigroup, define the binary operation on $M \times \Gamma(\Gamma \times M$, respectively) by

$(\forall (a, \alpha), (b, \beta) \in M \times \Gamma)\ (a, \alpha)(b, \beta) = (a \alpha b, \beta)$

$((\forall (\alpha, a), (\beta, b) \in \Gamma \times M)\ (\alpha, a)(\beta, b) = (\alpha, a\beta b)$ respectively),

then $M \times \Gamma$ ($\Gamma \times M$, respectively) is a semigroup (due to [4]). The semigroups $M \times \Gamma$ and $\Gamma \times M$ are called left and right operator semigroups of M, respectively.

Lemma. 13. (due to [4]) Let M be a regular Γ—semigroup, then M is an orthodox Γ— semigroup if and only if the set Reg $(M \times \Gamma)$ of regular elements of $M \times \Gamma$ are orthodox subsemigroups of $M \times \Gamma$ and $\Gamma \times M$, respectively.

Lemmal. 14. (due to [3]) Let M be a regular Γ—semigroup then for all $(a, \alpha) \in M \times \Gamma$, $V(a, \alpha) = \cup \{(x,\beta) : x \in V_\alpha^\beta(a)\}$.

$\beta \in \Gamma, V_\alpha^\beta(a) \neq \varnothing$

2, ORTHODOX Γ— REGULAR SEMIGROUP

According to Sen and Saha [2] a Γ—semigroup M is said to be an orthodox Γ—semigroup if M is a regular Γ—semigroup and $(E_\alpha\ \alpha\ E_\beta) \cup ((E_\beta\ \alpha\ E_\alpha) \subseteq E_\beta$ holds for all $\alpha, \beta \in \Gamma$. Γ—orthodox semigroup, i. e. its every related semigroup M_α. ($\alpha \in \Gamma$) is an orthodox semigroup, had been introduced and the following three examples are given by Zhao Xian Zhong in [3].

Example 2.1. Let Q* denote all non—zero rational numbers and Γ denote all positive integers. Define that $a\alpha b$ is the product of three numbers $|a|$, α, and b, then Q* is both an orthodox Γ—semigroup and a Γ—orthodox semigroup.

Example 2.2. Let M be the set of all positive integers. Suppose that $\Gamma = M$, define $a\alpha b = [a, \alpha, b]$ = the l. c. m of a, α and b, for all a, b $\in$ M and all $\alpha \in \Gamma$. Then M is an orthodox Γ—semigroup but M is not a Γ—orthodox semigroup.

Example 2.3. Suppose that $I = \Lambda = \{1,2\}$, $G = \{e,g,g^2,g^3\}$ is the cyclic group of order four generated by g, and

$$P = (P_{ij}) = \begin{pmatrix} g & g^3 \\ g^2, & e \end{pmatrix}, \quad Q = (q_{ij}) = \begin{pmatrix} g & g^2 \\ g^3, & e \end{pmatrix}$$

are two 2×2 matrices on G. Take $M = G \times I \times \Lambda$, $\Gamma = \{\alpha, \beta\}$ and define $(a, i, \lambda)\alpha(b, j, \mu) = (ap_{\lambda j}b, i, \mu)$ and $(a, i, \lambda)\,\beta\,(b, j, \mu) = (aq_{\lambda j}b, i, \mu)$, Then M is a Γ—orthodox semigroup, but M is not an orthodox Γ—semigroup.

It is illustrated by the above examples that the class of Γ—orthodox semigroups and the class of orthodox Γ—semigroups are not contained within each other, and the intersection of these two classes is not empty. Now we ask : what is this intersection? The answer will be given in the following.

Definition 2.4. A Γ—semigroup M is said to be an orthodox Γ—regular semigroup if M is an orthodox Γ—semigroup and its every related semigroup M_α. ($\alpha \in \Gamma$) is regular.

Proposition 2.5. A Γ—semigroup M is an orthodox Γ—regular semigroup if and only if M is both an orthodox Γ—semigroup and Γ—orthodox semigroup.

Proof : Omitted.

The following proposition 2.6 will be directly obtained from Lemma 1.3.

Proposition 2.6. A Γ—semigroup M is an othodox Γ—regular semigroup if and only if its right operator semigroup ΓxM and its left operator semigroup MxΓ are orthodox.

Proof : Omitted.

Lemma 2.7. If M is an orothodox Γ—regular semigroup, then $E_\alpha\,\beta\,E_\beta = E_\beta\,\beta\,E_\alpha = E_\alpha$ holds for all $\alpha, \beta \in \Gamma$.

Proof : For any $\alpha, \beta \in \Gamma$. and any $e \in E_\alpha$ there exists $x \in M$ such that $x \in V_\alpha^\beta(e)$ by lemma 1.1. This implies $e = e\alpha x\beta e$ and $e\alpha x \in E_\beta$, hence, $E_\alpha \subseteq E_\beta\beta E_\alpha$. Since M is an orthodox Γ—semigroup, we have $E_\beta\beta E_\alpha \subseteq E_\alpha$, and so $E_\alpha = E_\beta\beta E_\alpha$. The proof of $E_\alpha = E_\beta\beta E_\alpha$ is similar.

Lemma 2.8. Let M be a regular Γ—semigroup. If for all $\alpha, \beta \in \Gamma$, $E_\alpha\beta E_\beta = E_\beta\beta E_\alpha = E_\alpha$ holds, then M is an orthodox Γ—regular semigroup.

Proof : M is obviously an orthodox Γ—semigroup. To prove the remaining half, suppose that for any $\alpha, \beta \in \Gamma$, $E_\alpha\beta E_\beta = E_\beta\beta E_\alpha = E_\alpha$. Then for any (but fixed) $e \in E_\alpha$. there exist $x \in E_\alpha$ and $y \in E_\beta$ such that $x\beta y = e$. Hence in particular $e\beta y = e$ and $e = e\beta(y\alpha e) = e\beta(y\alpha e)\alpha e$. By $y\alpha e \in E_\beta\alpha E_\alpha = E_\beta$ we have that $(y\alpha e)\alpha e\beta(y\alpha e) = (y\alpha e)\beta(y\alpha e) = y\alpha e$. This implies $y\alpha e \in V_\beta^\alpha(e)$ and $V_\beta^\alpha(e) \neq \varnothing$.

Similarly, we have V_{α}^{β} (e) ≠ ∅. Now from lemma 1.1, it follows that M is a Γ—regular semigroup.

The following thorem follows from 2.7 and lemma 2.8.

Theorem 2.9. A regular Γ—semigroup is an orthodox Γ—regular semigroup if and only if for all α, β ∈ Γ, $E_\alpha \beta E_\beta = E_\beta \beta E_\alpha = E_\alpha$.

Let us now prove the following proposition which will provide the recipe for manufacturing the class of orthodox Γ—semigroups.

Proposition 2.10. Let (S, o) be an orthodox semigroup. Suppose that M = S, and Γ is a nonempty subset of S, and aαb is the product of three elements a, α, b in the semigroup (S, o). Then we have

(i) M is an orthodox Γ—semigroup.

(ii) If S contains the identity e and Γ is an $\mathcal{H}$—class containing e in S, then M is an orthodox Γ— regular semigroup.

Proof : (i) M is obviously a Γ—semigroup. To prove the remaining half, notice that for amy α, β ∈ Γ, a ∈ E_α, and b ∈ E_β, we have aα,αa, bβ, βb ∈ E(S), where E(S) denotes the set of idempotents of S. Since S is orthodox, we now find that (aαb) β(aαb) = ((aα) (bβ)) ((aα) (bβ)) b = (aαbβ)b = aα(bβb) = aαb. Similarly, (bαa)β(aαa) = bαa. Thus $E_\alpha \alpha E_\beta \subseteq E_\alpha$ and $E_\beta \alpha E_\alpha \subseteq E_\beta$. Hence M is an orthodox Γ—semigroup as required.

(iii) It is only needed that M is a Γ—regular semigroup, by (i). For any a ∈ M, there exist x ∈ M and α, β∈Γ such that a = aαxβa, since M is a regular Γ_ semigroup. Now, note that the $\mathcal{H}$ —class H_e is a subgroup in S and Γ = H_e., we have a = $a\delta\delta^{-1}\alpha x\beta\delta^{-1}\delta a = a\delta z\delta a$ for all δ∈Γ, where $z = \delta^{-1}\alpha x\beta\delta^{-1}$ and δ^{-1} is the inverse of δ in group H_e. Hence M_δ is regular for all δ∈Γ, that is, M is a Γ—regular semigroup as required.

3. ORTHOGROUP Γ—SEMIGROUPS

Let M be a Γ—semigroup. If for some α∈Γ, the related semigroup M_α (α∈Γ) is a group, then every related semigroup is a group (due to Sen [1]), and M is called a Γ—group. So the left operator semigroup Γ × M is a union of groups. Moreover, we can show that Γ × M is simple. Hence Γ × M is a completely simple semigroup. In this section we want to generalize these results.

Definition 3.1. A Γ—semigroup M is said to be an orthogroup Γ—semigroup if M is an orthodox Γ—regular semigroup and for some α∈Γ, the related semigroup M_α, is a union of groups.

It is clear that every Γ—group is an orthogroup Γ—semigroup. But it will be shown in example 3.7 that not every orthodox Γ—regular semigroup is an orthogroup Γ—semigroup.

Let M be a Γ—semigroup, if for some α∈Γ, the related semigroup M_α is a union of groups, M_β is not necessarily a union of groups for each β∈Γ. For example, suppose that Γ is the set $\{\alpha, \beta\}$, and M is a Γ—semigroup with the related semigroup M_α being a o—group $G \cup \{o\}$, and the related semigroup M_β being a null semigroup. Then M_α is a union of groups, but the M_β is not a union of groups. We prove that every related semigroup of an orthogroup Γ—semigroup is a union of groups. To do this the following results are needed.

Lemma 3.2 Let M be a Γ—regular semigroup and $M \times \Gamma$ denote its left operator semigroup. Then for all $(a, \alpha) \in M \times \Gamma$ we have

(i) $L(a, \alpha) = \{(x, \alpha) : x \in L_a\} = (L_a, \alpha)$,

(ii) $R(a, \alpha) = \{(y, \beta) : y \in R_a, \beta \in \Gamma\} = (R_a, \alpha)$,

(iii) $H(a, \alpha) = \{(z, \alpha) : z \in H_a\} = (H_a, \alpha)$

where $L(a, \alpha)$ ($R(a, \alpha)$, $H(a, \alpha)$) denotes L—class (R—class, H—class) containing the element (a, α) of $M \times \Gamma$, L_a (R_a, H_a) denotes L—class (R—class, H—class) containing the element a in M.

Proof. Omitted :

Lemma 3.3 Let M be an othodox Γ—regular semigroup, and $M \times \Gamma$denotes its left operator semigroup, and B denotes the set of idempotents of $M \times \Gamma$. Then we have

(i) $J^{\beta}_{(e, \alpha)} = \bigcup_{\beta \in \Gamma} (V^{\beta}_{\alpha}(e), \beta)$ for all $(e, \alpha) \in B$

(ii) For all $(e, \alpha), (f, \beta) \in B$, if $(f, \beta) \in R(e, \alpha)$, then $(f, \beta) \in J^{\beta}_{(e, \alpha)}$, where $J^{\beta}_{(e, \alpha)}$ denotes J—class containing (e, α) in B and $R(e, \alpha)$ denotes R—class containing (e, α) in $M \times \Gamma$.

Proof, (i) By Lemma 1.3 and Lemma 1.4 B is a band, and so $J^{\beta}_{(e, \alpha)} = V(e, \alpha)$ $= \bigcup_{\beta \in \Gamma} (V^{\beta}_{\alpha}(e), \beta)$, since M is Γ—regular semigroup.

(ii) If $(f, \beta) \in R(e, \alpha)$, then there exist (x, β) and $(y, \alpha) \in M \times \Gamma$ such that $(e, \alpha)(x, \beta) = (f, \beta)$, and $(f, \beta)(y, \alpha) = (e, \alpha)$. Hence in particular $e\alpha x = f$ and $f\beta y = e$. Moreover, $e\alpha f = f$ and $f\beta e = e$. Notice that $e \in E_\alpha$ and E_β, thus we have $f\beta e\alpha f = f$ and $e\alpha f\beta e = e$, that is, $f \in V^{\beta}_{\alpha}(e)$. By (i), $(f, \beta) \in J^{\beta}_{(e, \alpha)}$, as required.

Proposition 3.4. The left operator semigroup $M \times \Gamma$ of an orthogroup Γ—semigroup M is a union of groups.

Proof. Let M be an ortho group Γ—semigroup, and $M \times \Gamma$ denote its left operator semigroup, and B denote the set of idempotents of $M \times \Gamma$. Since M is a Γ—regular semigroup, every $\mathfrak{R}$—class of M contains at least one α—iedmpotent, say e, (i, e. $e\alpha e = e$) for all $\alpha \in \Gamma$. So every $\mathfrak{R}$—class of $M \times \Gamma$ contains at least one idempotent which belongs to the set $\{ (x, \alpha) : x \in M\}$ for all $\alpha \in \Gamma\}$ by lemma 3.2. Now, suppose that for some (but fixed) $\delta \in \Gamma$., the related semigroup M_δ is a union of groups, and (e, α), (f, β) ∈ B as well as (e, α) $\mathscr{D}$ (f, β) in M × Γ. As above there exists (c, δ) ∈ B such that (c, δ) ∈ $R_{(f,\ \beta)}$, and so (c, δ)J^B (f, β) by Lemma 3.8 (ii). Since M_δ is a union of groups, there exists (d, δ) ∈ B such that (d, δ) ∈ R (e, α) and (d, δ) ∈ L (c, δ). Hence (e, α) J^B (d, δ) by Lemma 3.4 (ii), and (d, δ) J^B (c, δ), M_δ being a union of groups. So (e, α) J^B (f, β), because M is an orthodox Γ semigroup, M × Γ is an orthodox semigroup by lemma 1.3. Now notice we have proved that (e, α), (f, β) ∈ B and (e, α) $\mathscr{D}$ (f, β) imply (e, α) J^B(f, β), thus M × Γ is a union of groups by proposition vl. 3.3 of Howie J.M. [6].

A Γ—semigroup M is said to be a Γ—union of groups if its every related semigroups is a union of groups.

Lemma 3.5 A Γ—semigroup M is a Γ—union of groups if and only if its left (right, respectively) operator semigroups is a union of groups.

Proof. Omitted

Now, the following theorem can be directly obtained by Lemma 3.4 and Lemma 3.5.

Theorem 3.6. A Γ—semigroup M is an orthogroup Γ—semigroup if and only if it is both an orthodox Γ—semigroup and a Γ—union of groups.

Proof. Omitted.

Finally, We shall illustrate that not every othodox Γ—regular semigroup is an orthogroup Γ—semigroup by the following example.

Example 3.7. let M be the direct product T_EXG of the bicyclic semigroup and a group G with |G|> 1. Suppose, the Γ is the $\mathscr{H}$—class containing the identity e of M, and aαb is the product of three elements a, α, b of the semigroup M for all, a, b ∈ M and all α∈Γ, then M is an orthodox Γ regular semigroup by proposition 2.10 (ii). But if α denotes the identity e of M, then the related semigroup M_α with the identity is not union of groups, so M is not an orthogroup Γ—semigroup.

4. RECTANGULAR Γ—GROUPS

We consider here orthodox completely simple Γ—semigroup. As we shall see they are quite special. In the context, the following concept plays a basic role.

A Γ—rectangular band means a Γ—semigroup such that its every related semigroup is rectangular band. Also it is clear that for all a, b ∈ M and all α, β∈Γ, aαb = aβb holds in a Γ—rectangular band M. So it is often convenient to say rectangular band instead of Γ—rectangular band.

Definition 4.1. A Γ—semigroup is said to be a rectangular Γ—group if it is isomorphic to the direct product of a rectangular band and a Γ—group.

By Seth A. [5] every completely simple Γ—semigroup admits a Rees matrix representation with a set of sandwitch matrices.

Proposition 4.2 The following conditions on M = M (G. I. Λ. P) are equivalent.

(i) M is an orthodox Γ—semigroup.

(ii) For any i, j $\in$ I, λ μ $\in$ Λ, P = $(P_{\lambda i})$, Q= $(q_{\mu j})$ $\in$ P.

$p_{\lambda i}^{-1}q_{\lambda i}^{-1}q_{\mu j}^{-1}p_{\mu i}$ = e and $q_{\mu j}^{-1}q_{\mu j}p_{\lambda i}^{-1}p_{\lambda i}$ = e

where e is the identity of G.

Proof : Firstly, notice that $E_p = \{(p_{\lambda i}^{-1}, i, \lambda); i \in I, \lambda \in \Lambda\}$ and $E_Q = \{(q_{\mu i}^{-1}, j, u) : j\in I, \mu\in \Lambda\}$. For any $(q_{\lambda i}^{-1}, i, \lambda) \in E_P$ and any $(q_{\mu i}^{-1}, j, \mu) \in E_Q$, we have that

$(p_{\lambda i}^{-1}, i, \lambda)\ Q(q_{\mu i}^{-1}, j, \mu) = (p_{\lambda i}^{-1}\ q_{\lambda l}, (q_{\mu i}^{-1}, i, \mu) \in E_P$ and $(q_{\mu i}^{-1}, j, \mu)\ Q(q_{\lambda i}^{-1}, i, \lambda) = (q_{\mu j}^{-1}\ q_{\mu i}\ p_{\lambda i}^{-1}\ j, \lambda) \in E_Q$ if and only if $p_{\mu i}^{-1}\ q_{j\mu}^{-1}\ q_{\mu j}^{-1}\ p_{\mu j}^{-1} = q_{\mu j}^{-1}\ q_{\mu j}\ p_{\lambda j}^{-1}\ p_{\lambda j} = e$.

Thus (i) if and only if (ii) holds.

Theorem 4.3. A Γ—semigroup M is an orthodox completely simple Γ —semigroup if and only if it is a rectangular Γ —group.

Proof : A rectangular Γ —group is obviously orthodox completely simple Γ —semigroup by proposition 4.2. Suppose that M is an Orthodox completely simple Γ —semigroup. Given $\alpha\in\Gamma$, by theorem 2.5. of J. M. Howise [6] $\varnothing : H_n \times I \times \Lambda \to M$ given by $(a,i, \lambda)\varnothing = e_i\alpha a\alpha f_\lambda$ is an isomorphism from the Rees matrix semigroup M $[H_n ; I \Lambda ; P]$ onto M_α, where e_i and f_λ are α—identity of H_{i1} and $H_{1\lambda}$ respectively. Since M_α is a union of groups for all $\alpha\in\Gamma$, every $\mathscr{H}$—class in M is a sub Γ—group of M if we define $(i, \lambda)\ \beta(j, \mu) = (i, \mu)$ for all $(i, \lambda) \in I \times \Lambda$ and all $\beta\in\Gamma$, then $I \times \Lambda$ becomes a rectangular band. So $H_n \times I \times \Lambda$ is a rectangular Γ—group. Now we show that φ is an isomorphism from $H_n \times I \times \Lambda$ onto Γ—semigroup M. Since for any (a,j,λ), (b,i,μ), $\in H_n \times I \times \Lambda$ and, $\beta\in\Gamma$,

we have

$((a,i,\lambda)\ \beta\ (b,i,\mu)\varphi$

$= (a\beta b,\ i,\mu)\varphi$

$= e_j\alpha(a,\beta b)\alpha f_\mu$

$= (e_i\alpha a)\alpha(I_\alpha\beta I_\beta)\ \beta b\alpha f_\mu$ ($I_\alpha$and 1_β being α—indentity and β—indentity in Γ—group H_n respectively)

$= (e_i\alpha a)\alpha(I_\alpha\alpha f_\lambda\beta I_\beta)\ \beta b\alpha f_\mu$

$= (e_i\alpha a)\alpha(f_\lambda\beta I_\beta)\ \beta b\alpha f_\mu$

$= (e_i\alpha a)\alpha(f_\lambda\beta I'_\beta\ \beta I_\beta)\ \beta b\alpha f_\mu$ (I'_β being β—indentity in H_n)

$= (e_i\alpha a\alpha f_\lambda)\beta(I'_\beta\ \beta I_\beta)\ \beta b\alpha f_\mu$

$= (e_i\alpha a\alpha f_\lambda)\beta(I'_\beta\ \beta e_i\alpha I_\beta)\ \beta b\alpha f_\mu$

$= (e_i\alpha a\alpha f_\lambda)\beta(e_i\alpha b\alpha f_\mu)$

$= (a,\ i,\ \lambda)\ \varphi\ \beta(b,\ j,\ u)\varphi$, as required.

Theorem 4.4. An orthogroup Γ—semigroup is a semilattice of rectangular Γ—groups.

Proof : An orthogroup Γ—semigroup M is a Γ—union of groups. It means its every related semigroup $M_\alpha(\alpha \in \Gamma)$ is a union of groups. So we can say the Green's relations J on M_α (for all $\alpha \in \Gamma$) are same, and M is a semilattice of completely simple Γ—semigroups. Moreover, note that every related semigroup M_α of M is orthodox. So we have M is a semilattice of Orthodox completely simple Γ—semigroup. Hence M is a semilattice of rectangular Γ—groups by theorem 4.3.

The authors would like to thank Professor Guo Y. Q. for his kind help and encouragement for the preparetion of this paper.

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